Date:- 16.08.2025 Chapter-1 Force.

Q. Define Force.

Ans:- It is an agent which produces or tends to produce, destroy or tends to destroy the motion of a body, the agent is called a force.

Types of Force.

Ans:- There are different types of forces. They are:-

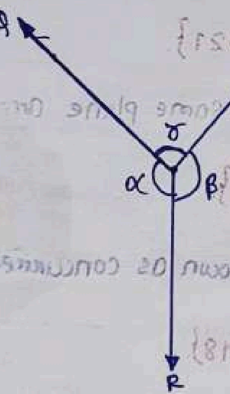
- Coplaner Forces:- {2006, 2012, 2017(B), 2019, 2021}.
  The forces whose lines of action lie on the same plane are known as Co-planar forces.
- Concurrent Forces:- {2008, 2011, 2014(B), 2017}.
  The forces which meet at one point are known as concurrent Forces.
- Collinear Forces:- {2007, 2009, 2013(B), 2016, 2018}.
  The forces whose lines of action lie on the same line are known as Collinear Forces
- Sub type:- Coplanar concurrent forces:- {2007, 2008, 2010(B)}.
  The forces which meet at one point and their lines of action also lie on the same plane are known as coplaner concurrent forces.
- Coplaner Non-concurrent forces:- {2012(B), 2017}
  The forces which do not meet at one point but their lines of action lie on the same plane are known as Coplaner Non-concurrent Forces.
- Non-coplaner Concurrent Force:- {2006, 2011(B), 2015(B), 2018(B)}.
  The forces which meet at one point but their lines of action do not lie on the same plane are known as Non-coplaner Concurrent Forces.
- Non-coplaner non-concurrent force:- {2005, 2008(B), 2013, 2017}.
  The forces which do not meet at one point and their lines of action do not lie on the same plane are known as non-coplaner nonconcurrent forces.
- Resultant Force:- {2007, 2010, 2012(B), 2015, 2017, 2019(B), 2021}.
  If a no. of force P, Q, R are acting simultaneously on a particle then it is possible to find out a single force which would produced the same effect as produce by all the given forces, the single force is called Resultant Force.
- Composition of Forces:- {2008, 2009, 2012, 2014, 2018, 2021}.
  The process of finding out the resultant force of a no. of given forces is called as composition of forces.
- Resolution of Force:- {2006, 2007(B), 2009, 2011, 2012(B), 2013, 2014, 2016, 2018(B), 2019(B), 2020, 2020(B)}.
  The process of splitting up the given force into a no. of components without changing

its effect on the body is called Resolution of Force.

* **Lame's Theorem**:- {2006, 07(B), 08, 10, 12, 12(B), 13, 13(B), 14, 14(B), 15, 16, 18, 19(B), 20, 21}.

If three co-planer forces are acting at a point be in equilibrium then each force is proportional to the sine of angle between the other two forces.

Mathematically,



$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

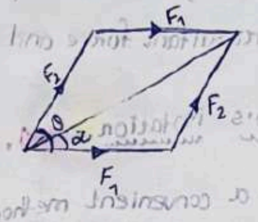
where, P, Q, R are the three forces α, β, γ are the three angles as shown in the figure.

Date :- 20.03.2025

* **Parallelogram law of Forces** {2007, 11, 13, 14, 17, 20}.

If two forces acting simultaneously on a particle be represented in magnitude and direction by the two adjacent sides of a parallelogram then their resultant may be represented in magnitude and direction by the diagonal of the llgm which passes through their point of intersection.

Mathematically,



Resultant force,

i.e.

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

and $\tan\alpha = \dfrac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

where, F_1 and F_2 = the forces which resultant is required to be found out.

θ = the angle between the forces F_1 & F_2

α = the angle which resultant force makes with any one of the forces {F_1 & F_2}.

* **Conditions of Equilibrium of a system of Co-planar forces** :- {2006, 2011, 2015, 2018}.

The condition of equilibrium of a system of co-planar forces are

- The horizontal components of all the forces is zero, i.e. $\Sigma H = 0$.
- The vertical components of all the forces is zero, i.e., $\Sigma V = 0$.
- The resultant moment of all the forces is zero, i.e. $\Sigma M = 0$.

* **Triangle law of Forces** :- {2008, 2011, 2012, 2014, 2015, 2017, 2018, 2020}.

If two forces acting simultaneously on a particle be represented in magnitude & direction by the two sides of a triangle taken in order then their resultant may be represented in magnitude and direction by the third side of the △ taken in opposite order.

* **Polygon Law of Forces** :- {2005, 2013, 2015, 2017}.

If a numbers of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order then the resultant of these forces may be represented in magnitude & direction by the closing side of the polygon taken in opposite order.

* Space Diagram or Position Diagram:- {2007, 2012, 2014, 2018}.

Space diagram is the construction of a diagram showing the various forces or various loads along with their magnitude and lines of action.

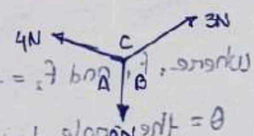
* Vector Diagram/Force Diagram:- {2007, 2012, 2014, 2018}.

Vector diagram is the construction of a diagram starting from a convenient point and then go on adding all the forces vectorially one by one, to some suitable scale, the closing side of the polygon taken in opposite order will give the magnitude of the resultant force and the direction.

* Bow's Notation:- {2010, 2011, 2012, 2014, 2017}.

It is a convenient method in which every force or every load is named by two capital letters placed on its either side in the space diagram.

Acc. to Bow's Notation, from the figure we have 2N Force name is AB, 3N force name BC, 4N force name is CA.



* Law of Transmisibility of Forces:- {2010B, 2016}.

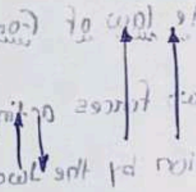
If a force act at any point on the rigid body then it may also be considered to act at any other point on its lines of action, provided this point is rigidly connected with the body.

* Like ll Forces:-

The forces which are ll to each other and having same direction

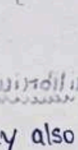
* Unlike ll Forces:-

The forces which are ll to each other and having opposite direction.

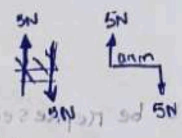


* Equal Forces:-

The forces which are ll to each other & having same magnitude



* Couple:-



{2006, 2008, 2009, 2017, 2019 B, 2021}.

A pair of two equal and unlike parallel forces i.e., the forces equal in magnitude & lines of action ll to each other and acting in opposite direction is known as couple.

E.g.:- Key of a lock while locking or unlocking.

* **Free Body Diagram:-** {2011, 2015B}.

The diagram of the isolated elements or a portion of the body alongwith the net effect of the system on it is called a free body diagram.

* **D-Alemberts Principle:-** {2005, 2010, 2012-B, 2013, 2013 B, 2014, 2017, 2018 B, 2020}.

If a rigid body is acted upon by a system of forces then, these system may be reduced to a single resultant force whose magnitude, direction and lines of action may be found out by the method of graphics statics.

* **Centrifugal Force:-** {2011, 2014}.

The force which always tends to throw the body away from the centre of a circular path is called centrifugal force.

* **Centripetal Force:-**

The Force which act along the radius of the circle at every point and is always directed towards the centre of the circle, along which the body moves is called centripetal force.

* **Methods of Resolution of the Resultant Force:-**

- Resolving all the forces horizontally i.e., ΣH.
- Resolving all the forces vertically i.e., ΣV.
- Magnitude of the Resultant force i.e., $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$
- Direction of Resultant Force. i.e., $\tan\theta = \frac{\Sigma V}{\Sigma H}$

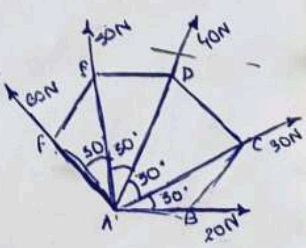
$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

Date:- 23.03.2025

Numericals.

Type:- Resolution of Resultant Force

Q1. The forces 20N, 30N, 40N, 50N and 60N are acting at 1 of the angular points of a regular hexagon towards the other 5 angular points, taken in order. Determine the mag. and direction of the resultant force.



Sol^n. Resolving all the forces horizontally, $\Sigma H = 20 \times \cos 0° + 30 \times \cos 30° + 40 \times \cos 60° + 50 \cos 90° + 60 \times \cos 120°$.

$=$

$= 36N.$

Force x cos θ.

Resolving all the forces vertically, $\Sigma V = 20\sin 0° + 30\times\sin 30° + 40\times\sin 60° + 50\times\sin 90° + 60\times\sin 120°$.

$= 151.6\,N.$

Magnitude of the resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{36^2 + 151.6^2}$

$= 155.8\,N.$

Direction of Resultant force,

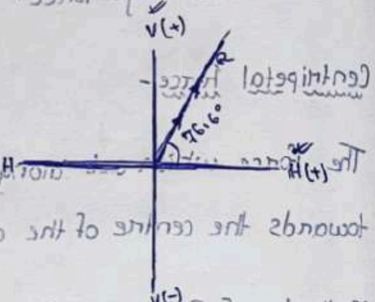
Let, θ = the angle which resultant force makes with the horizontal.

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H}$$

$$= \tan^{-1}\frac{151.6}{36}$$

$$\theta = 76.6°$$



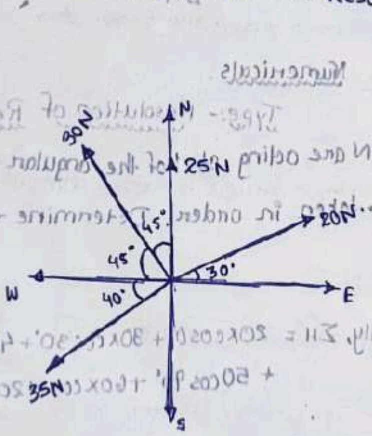
Date:- 27.03.2025

Q2. The following forces are acting at a point

- 20 N. inclined at 30° towards north of east
- 25N towards north.
- 30N towards north west.
- 35N inclined at 40° towards south of west.

Determine the magnitude and direction of the resultant force.

Soln.



Resolving all the forces horizontally,

$$\Sigma H = 20\cos 30^\circ + 25\cos 90^\circ + 30\cos 135^\circ + 35\cos 220^\circ$$

$$= -30.7\ N.$$

Resolving all forces vertically,

$$\Sigma V = 20\sin 30^\circ + 25\sin 90^\circ + 30\sin 135^\circ + 35\sin 220^\circ$$

$$= 33.7\ N$$

Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$= \sqrt{(-30.7)^2 + (33.7)^2}$$

$$= 45.6\ N$$

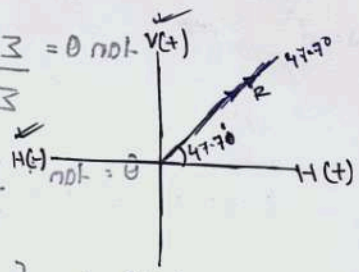
Direction of the resultant force,

let, θ = the angle which the resultant force makes with the horizontal.

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{33.7}{-30.7}\right)$$

$$= 47.7^\circ$$

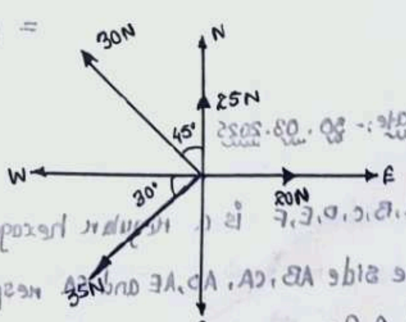


∴ Actual angle, $\theta = 180^\circ - 47.7^\circ$

$$\theta = 132.3^\circ$$

Q3. A particle is acted upon by the following forces.

- 20N in the direction due east.
- 25N in the direction due north.
- 30N in the direction north-west.
- 35N in the direction 30° south of west.



Determine the mag. and direction of the resultant force.

• Resolving all the forces horizontally, $\Sigma H = 20\cos 0^\circ + 25\cos 90^\circ + 30\cos 135^\circ + 35\cos 210^\circ$

$= -31.52\,N$

$= 31.52\,N$ {Force can't be -'ve}.

• Resolving all the forces vertically, $\Sigma V = 20\sin 0^\circ + 25\sin 90^\circ + 30\sin 135^\circ + 35\sin 210^\circ$

$= 28.71\,N$

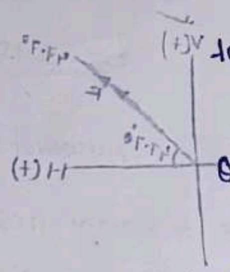
• Magnitude of Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(31.52)^2 + (28.71)^2}$

$= 42.63\,N$

• Direction of the resultant force.

Let, θ = the angle which resultant force makes with the horizontal.

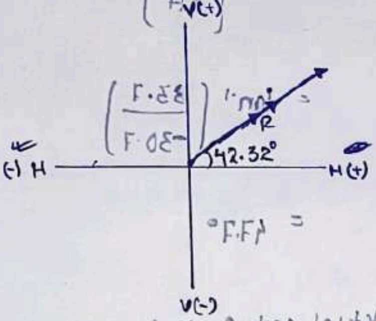


$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\dfrac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left[\dfrac{28.71}{31.52}\right]$

$\theta = 42.32^\circ$

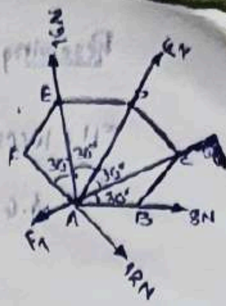


$\therefore$ Actual angle $= 180^\circ - 42.32^\circ$

$= 137.68^\circ$

Date:- 30.03.2025

4. A, B, C, D, E, F is a regular hexagon. The forces 8N, F_1, F_2, 16N and 12N, acting along the side AB, CA, AD, AE and FA respectively and are in equilibrium. Determine the magn of F_1 & F_2.



Resolving all the forces horizontally,

$$\Sigma H = 8\cos 0^\circ - F_1 \cos 30^\circ + F_2 \cos 60^\circ + 16\cos 90^\circ - 12\cos 120^\circ$$

$$= 14 - F_1 \frac{\sqrt{3}}{2} + F_2 \frac{1}{2}$$

$$= 14 - F_1 \times 0.8 + F_2 \times 0.5 \longrightarrow (1)$$

Resolving all the forces vertically,

$$\Sigma V = 8\sin 0^\circ - F_1 \sin 30^\circ + F_2 \sin 60^\circ + 16\sin 90^\circ - 12\sin 120^\circ$$

$$= 0 - F_1 0.5 + 0.8F_2 + 16 - 10.4$$

$$= 5.6 - 0.5F_1 + 0.8F_2 \longrightarrow (11)$$

By the given condition of equilibrium of forces,

WKT,

$\Sigma H = 0$

$14 - 0.8F_1 + 0.5F_2 = 0 \longrightarrow (1)$

$\Sigma V = 0$

$5.6 - 0.5F_1 + 0.8F_2 = 0 \longrightarrow (11)$

Now,

$-0.8F_1 + 0.5F_2 = -14 \quad \times 0.5$

$-0.5F_1 + 0.8F_2 = -5.6 \quad \times 0.8$

$F_1 0.4 + 0.25F_2 = 7$

$0.4F_1 + 0.64F_2 = 4.48$

$-0.39F_2 = 2.52$

$F_2 = \frac{2.52}{0.39}$

$F_2 = 6.4N$

$F_2 = 6.4N$ {force can't be (-)ve}.

Putting the value of F_2 in eqn no. (1)

$0.8F_1 + 0.5F_2 = 14$

$0.8F_1 + 3.2 = 14$

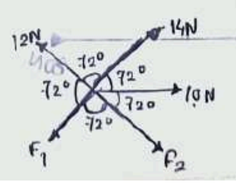
$0.8F_1 = 10.8$

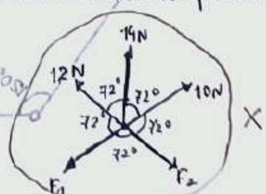
$F_1 = 13.5N$

Ans. $F_1 = 21.5N$

$F_2 = 6.4N$.

Q5. Five string are tied at a point and are pulled in all directions equally spaced from one another. If the mag. of pull on 3 consecutive strings are 10N, 14N & 12N respectively. Find the mag. of pull on other two strings.





Resolving all the forces horizontally,

$\Sigma H = 10\cos 0^\circ + 14\cos 72^\circ + 12\cos 144^\circ + F_1\cos 216^\circ + F_2\cos 288^\circ$

$= 4.61 - 0.8F_1 + 0.3F_2$

Resolving all the forces vertically,

$\Sigma V = 10\sin 0^\circ + 14\sin 72^\circ + 12\sin 144^\circ + F_1\sin 216^\circ + F_2\sin 288^\circ$

$= 20.36 - F_1 0.5 - 0.95F_2 \longrightarrow$

By the given condition of equilibrium of forces,

WKT,

$\Sigma H = 0$

$\Rightarrow 4.61 - 0.8F_1 + 0.3F_2 = 0$

$\Rightarrow \cancel{-}0.8F_1 + 0.3F_2 = \cancel{-}4.61 \longrightarrow (i) \times 0.5$

$\Rightarrow \Sigma V = 0$

$\Rightarrow 20.36 - F_1 0.5 - 0.95F_2 = 0$

$\Rightarrow \cancel{-}F_1 0.5 - 0.95F_2 = \cancel{-}20.36 \longrightarrow (ii) \times 0.8$

$0.4F_1 + 0.15F_2 = 2.305$

$0.4F_1 - 0.76F_2 = 16.288$

$(-) \quad (+) \quad (-)$

$F_2\, 0.91 = -13.983$

$F_2 =$

$0.4F_1 + 0.15F_2 = 2.305$

$0.4F_1 \pm 0.76F_2 = 16.288$

$(-) \quad (+) \quad (-)$

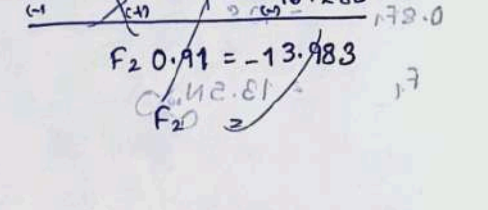
$0.91F_2 = -13.983$

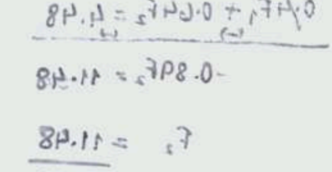
$F_2 =$

Ans.:- $F_1 = 28N$

$F_2 = 101.4$

Q8. A push force of 80N and a pull force of 70N acting simultaneously at a point. Find the mag. & direction of the resultant force, if the angle between them is 120°.





Ans.:- $R \rightarrow 128.9N$

$\theta \rightarrow 152.3^\circ$

• Resolving all the forces horizontally,

$$\Sigma H = -80\cos 0^\circ + 70\cos 120^\circ$$

$$= -80 - 35$$

$$= -115\,N = 115\,N \text{ (since force can't be (-ve))}$$

• Resolving all the forces vertically,

$$\Sigma V = -80\sin 0^\circ + 70\sin 120^\circ$$

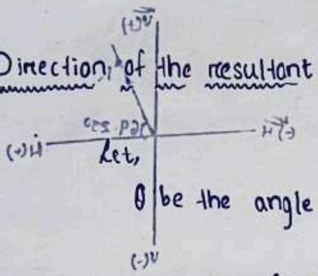
$$= 60.62\,N$$

• Magnitude of resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$= \sqrt{(-115)^2 + (60.62)^2}$$

$$= 129.9\,N.$$

• Direction of the resultant force,



Let,

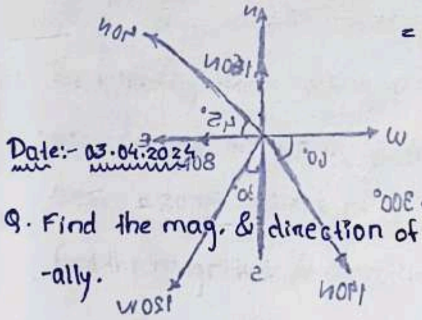
θ be the angle which resultant force makes with the horizontal.

$$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left[\frac{60.62}{115}\right]$$

$\therefore$ Actual angle $= 180^\circ - 27.47^\circ$

$$= 152.5^\circ$$



Date:- 03.04.2024

Q. Find the mag. & direction of the resultant force of the following forces acting horizontally.

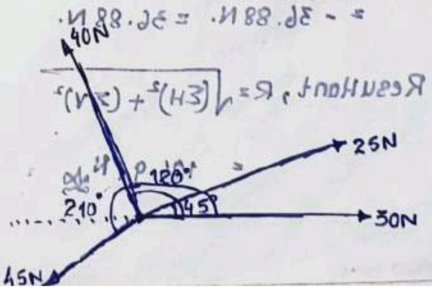
• 30N acting horizontally. • 25N, 40N, 45N acting respectively at 45°, 120°, 210° from the horizontal. The angles are measured anti-clockwise.

Soln,

Resolving all the forces horizontally,

$$\Sigma H = 30\cos 0^\circ + 25\cos 45^\circ + 40\cos 120^\circ + 45\cos 210^\circ$$

$$= 11.29\,N$$



Resolving all the forces vertically,

$\Sigma V = 30\sin 0^\circ + 25\sin 45^\circ + 40\sin 120^\circ + 45\sin 210^\circ$

$= 29.81N.$

Ans.

$R = 31.8N$

$\theta = 110.7^\circ$

Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= 31.8N$

• Direction of the resultant force:-

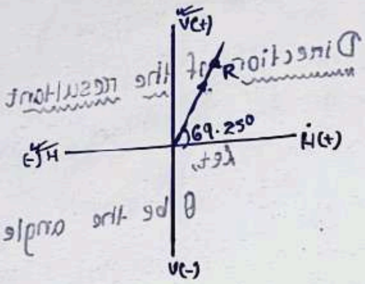
Let,

θ be the angle which resultant force makes with the horizontal.

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\frac{29.81}{11.29}$

$\theta = 69.25^\circ$



∴ Actual angle $= 180^\circ - 69.25^\circ$

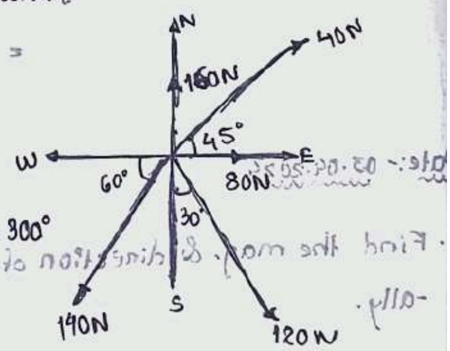
$= 110.7^\circ$

8. Find the mag. and direction of the resultant force for the following force system:

• 80N towards east. • 40N towards North-East. • 160N towards North.

• 140N inclined at 60° towards south of west • 120N inclined at 30° towards east of south.

Sol^n,



* Resolving all the forces horizontally.,

$\Sigma H = 80\cos 0^\circ + 40\cos 45^\circ + 160\cos 90^\circ + 140\cos 240^\circ + 120\cos 300^\circ$

$= 98.28N.$

* Resolving all the forces vertically,

$\Sigma V = 80\sin 0^\circ + 40\sin 45^\circ + 160\sin 90^\circ + 140\sin 240^\circ + 120\sin 300^\circ$

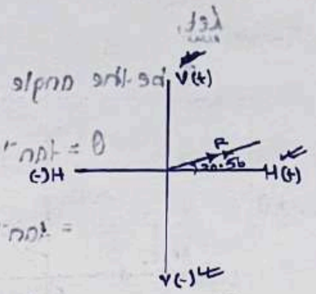
$= -36.88N. = 36.88N.$

* Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= 104.97N$

- **Direction of the resultant force:-**

Let, θ be the angle which resultant makes with the horizontal.



$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H}$$

$$\theta = \tan^{-1}\frac{36.88}{98.28}$$

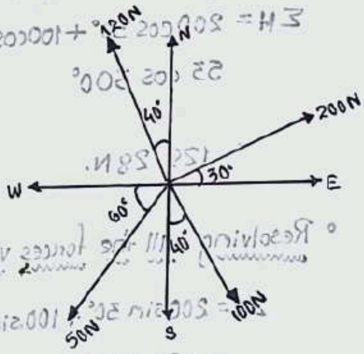
$$\theta = 20.56^\circ$$

9. A system of four co-planar forces acting on a body as given below:-

- 200N acting 30° North of East.
- 120N acting 40° West of North.
- 50N acting 60° South of west.
- 100N acting 40° East of South.

Determine the resultant force.

$\boxed{R = 153.12N}$



Sol^n,

- Resolving all the forces Horizontally,

$$\Sigma H = 200\cos 30^\circ + 120\cos 130^\circ + 50\cos 240^\circ + 100\cos 310^\circ$$
$$= 135.34N$$

- Resolving all the forces Vertically,

$$\Sigma V = 200\sin 30^\circ + 120\sin 130^\circ + 50\sin 240^\circ + 100\sin 310^\circ$$
$$= 72.01N$$

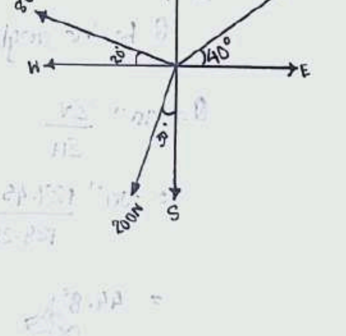
- Magnitude of the Resultant force,

$$R = \frac{\Sigma V}{\Sigma H} = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 153.3N$$

10. The following forces acting at a point:-

- 600N inclined at 40° towards North of East.
- 800N inclined at 20° towards North of West.
- 200N inclined at 30° towards west of south.

Find the magnitude & direction of the resultant force.



Soln:-

- Resolving all the forces horizontally,

$$\Sigma H = 600\cos 40^\circ + 800\cos 160^\circ + 200\cos 240^\circ$$
$$= -392.12N.$$

- Resolving all the forces vertically,

$$\Sigma V = 600\sin 40^\circ + 800\sin 160^\circ + 200\sin 240^\circ$$
$$= 486.08N.$$

R = 624.4N

Ans:- $\theta = 128.9^\circ$

• Magnitude of the Resultant force, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 624.5N$

• Direction of the Resultant force,

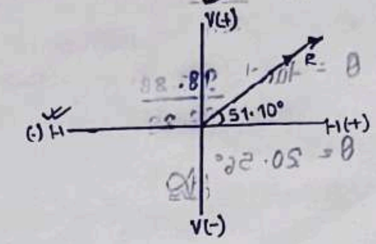
Let,

θ be the angle which resultant makes with the horizontal.

$$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H}$$

$$= \tan^{-1}\frac{486.08}{392.12}$$

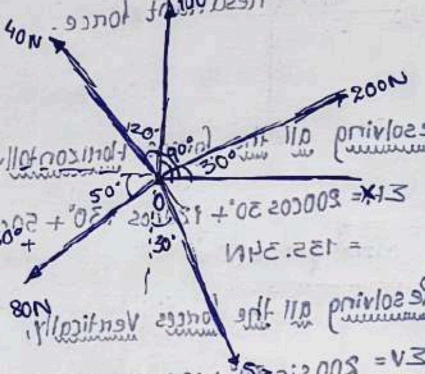
$$= 51.10^\circ$$



$\therefore$ Actual Angle $= 180^\circ - 51.10^\circ$

$= 128.90^\circ$

11. Five forces of mag. 200N, 100N, 40N, 80N and 53N are acting at a point on a body making an angle 30°, 90°, 120°, 230° & 300° respectively with the horizontal OX axis. All angles are measured anticlockwise. Find the mag. & direction of the resultant force.



Soln,

* Resolving all the forces horizontally,

$$\Sigma H = 200\cos 30^\circ + 100\cos 90^\circ + 40\cos 120^\circ + 80\cos 230^\circ + 53\cos 300^\circ$$

$$= 128.28N.$$

• Resolving all the forces vertically,

$$\Sigma V = 200\sin 30^\circ + 100\sin 90^\circ + 40\sin 120^\circ + 80\sin 230^\circ + 53\sin 300^\circ$$

$$= 127.45N.$$

• Magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= 180.8N$$

• Direction of the resultant force,

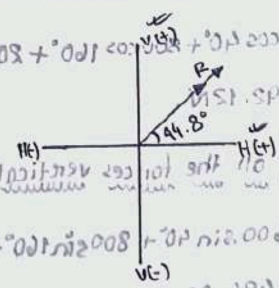
Let,

θ be the angle which resultant makes with the horizontal.

$$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H}$$

$$= \tan^{-1}\frac{127.45}{128.28}$$

$$= 44.8^\circ$$



12. Four horizontal wire are attached to a vertical post and they exert the following pulled on the post.

- 20KN, due North.
- 40KN due South-West.
- 30KN due east.
- 50KN due South-East.

Find the mag. & direction of the resultant force.

$R = 57.1KN$

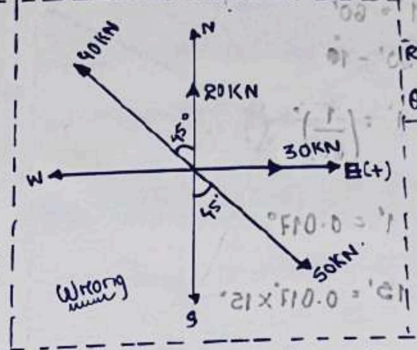
$\theta = 49.60.$

Sol^n,

- Resolving all the forces horizontally,

$\Sigma H = 30\cos 0^\circ + 20\cos 90^\circ + 40\cos 135^\circ + 50\cos 315^\circ$

$= 37.07^\circ.$



- Resolving all the forces vertically,

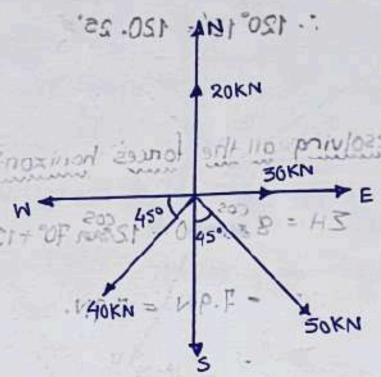
$\Sigma V = 30\sin 0^\circ + 20\sin 90^\circ + 40\sin 135^\circ + 50\sin 315^\circ$

$= 12.928^\circ.$

- Magnitude of resultant force,

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$=$



Sol^n,

- Resolving all the forces horizontally,

$\Sigma H = 30\cos 0^\circ + 20\cos 90^\circ + 40\cos 225^\circ + 50\cos 315^\circ$

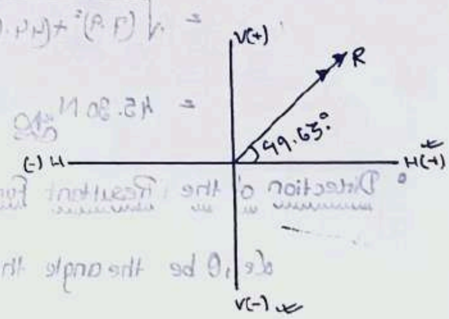
$= 37.01N.$

- Resolving all the forces vertically,

$\Sigma V = 30\sin 0^\circ + 20\sin 90^\circ + 40\sin 225^\circ + 50\sin 315^\circ$

$= -43.63N = 43.63N$

- Magnitude of the Resultant Force,



$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(37.01)^2 + (43.63)^2}$

$= 57.21N$

- Direction of the Resultant Force,

Let, θ be the angle that resultant makes with the horizontal.

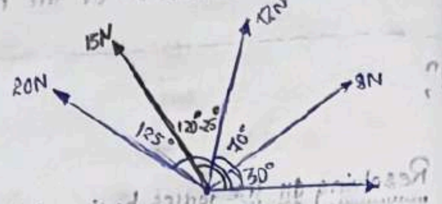
$\theta = \tan^{-1} \frac{\Sigma V}{\Sigma H}$

$= \tan^{-1} \frac{43.63}{57.01} = 49.63^\circ$

$\therefore \theta = 49.63^\circ$

13. Find the mag. & direction of the resultant force of the concurrent forces of 8N, 12N, 15N and 20N making an angle of 30°, 70°, 120° 15' and 125° respectively from a fixed line.

Conversion:-



$1^\circ = 60'$

$\Rightarrow 60' = 1^\circ$

$\Rightarrow 1' = \left(\frac{1}{60}\right)^\circ$

$\Rightarrow 1' = 0.017^\circ$

$\Rightarrow 15' = 0.017 \times 15^\circ$

$\Rightarrow 15' = 0.25^\circ$

$\therefore 120^\circ 15' = 120.25^\circ$

- Resolving all the forces horizontally,

$\Sigma H = 8\cos 30^\circ + 12\cos 70^\circ + 15\cos 120.25^\circ + 20\cos 125^\circ$

$= -7.9N = 7.9N.$

- Resolving all the forces vertically,

$\Sigma V = 8\sin 30^\circ + 12\sin 70^\circ + 15\sin 120.25^\circ + 20\sin 125^\circ$

$= 44.61N$

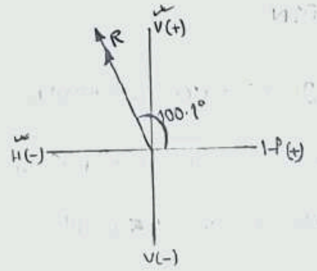
R ≈ 45.3N

θ ≈ 100.2°

- Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(7.9)^2 + (44.61)^2}$

$= 45.30\ N$



- Direction of the Resultant Force,

Let, θ be the angle that the resultant makes with the horizontal.

$\therefore \tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H}$

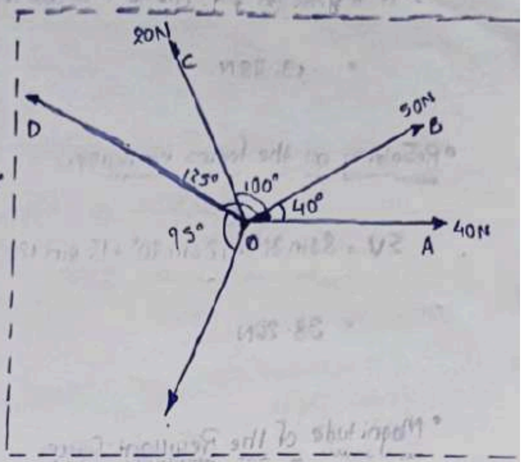
$= \tan^{-1}\frac{44.61}{7.9}$

$= 79.95^\circ$

$\therefore$ Actual Angle, $\theta = 180^\circ - 79.95^\circ$

$\theta = 100.1^\circ$

14. Four forces of mag. 40N, 50N, 20N and 60N are acting at a point O along a straight line OA, OB, OC, and OD respectively such that $\angle AOB = 40°$, $\angle BOC = 100°$, $\angle COD = 125°$ and $\angle DOA = 95°$. Find the mag. & direction of the resultant force.



- Resolving all the forces horizontally,

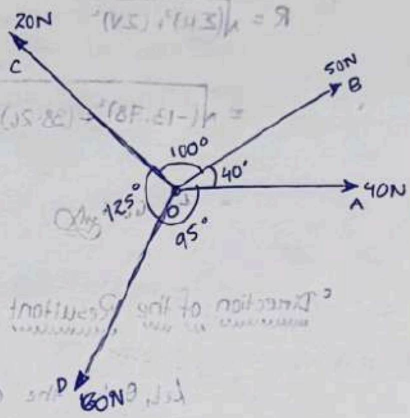
$\Sigma H = 40\cos 0° + 50\cos 40° + 20\cos 140° + 60\cos 265°$

$= 57.7N$.

- Resolving all the forces vertically,

$\Sigma V = 40\sin 0° + 50\sin 40° + 20\sin 140° + 60\sin 265°$

$= -14.7N$



- Resultant, $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(57.7)^2 + (14.7)^2}$

$= 59.5N$ Ans

- Direction of the Resultant force,

Let, θ be the angle that the resultant makes with the horizontal.

$\therefore \tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\frac{+14.7}{57.7}$

$\theta = 14.29°$ Ans

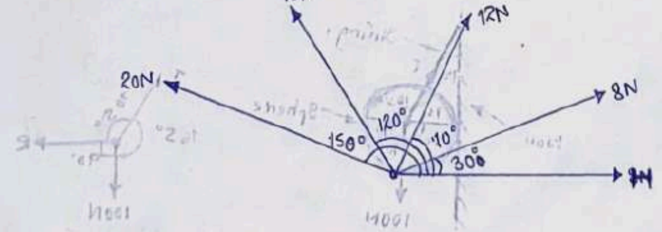
θ≈14.2°

15. Find the mag. and direction of the resultant force of the concurrent forces of 8N, 12N, 15N & 20N making an angle of 30°, 70°, 120° and 150° respectively from a fixed line.

R≈40.5N

θ= 109.8°

Sol^n,



° Resolving all the forces horizontally;

$$\Sigma H = 8\cos 30^\circ + 12\cos 70^\circ + 15\cos 120^\circ + 20\cos 150^\circ$$

$$= -13.78N.$$

° Resolving all the forces vertically,

$$\Sigma V = 8\sin 30^\circ + 12\sin 70^\circ + 15\sin 120^\circ + 20\sin 150^\circ$$

$$= 38.26N$$

° Magnitude of the Resultant Force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

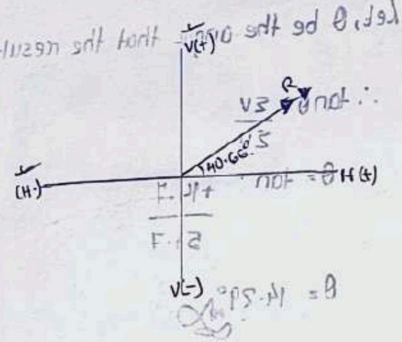
$$= \sqrt{(-13.78)^2 + (38.26)^2}$$

$$= 40.66N$$

° Direction of the Resultant Force,

Let, θ be the angle that the Resultant makes with the horizontal.

$$\therefore \tan\theta = \frac{\Sigma V}{\Sigma H}$$



$$\theta = \tan^{-1}\frac{38.26}{-13.76}$$

$$\theta = 70.21^\circ$$

$$\therefore \text{Actual Angle}, \theta = 180^\circ - 70.21^\circ$$

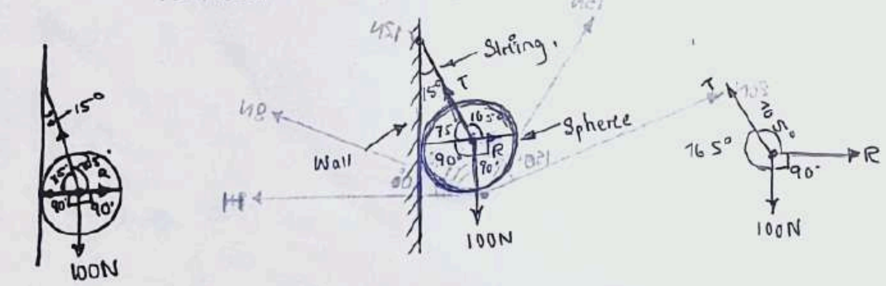
$$= 109.79^\circ$$

Date:- 10.04.2025

Type:- Lami's Theorem.

16. A sphere of weight 100N is fixed to a vertical wall by a string. The string makes an angle of 15° with the wall. Find the tension in the string and reaction from the wall.

Sol^n:-



Let,

$T \rightarrow$ Tension in the string.

$R \rightarrow$ Reaction from the wall.

Acc. to Lami's Theorem, from the fig we have.

$$\frac{R}{\sin 165^\circ} = \frac{T}{\sin 90^\circ} = \frac{100}{\sin 105^\circ}$$

Either

$$\frac{R}{\sin 165^\circ} = \frac{100}{\sin 105^\circ}$$

$$\Rightarrow R = \frac{100 \sin 165^\circ}{\sin 105^\circ}$$

$$\Rightarrow R = 26.7N$$

Or

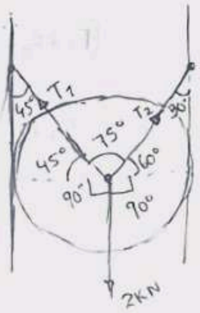
$$\frac{T}{\sin 90^\circ} = \frac{100}{\sin 105^\circ}$$

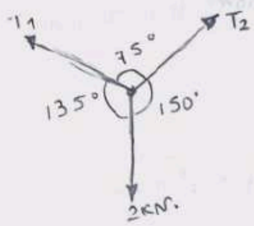
$$\Rightarrow T = \frac{100 \times \sin 90^\circ}{\sin 105^\circ}$$

$$\Rightarrow T = 103.5N$$

17. Two men carry a weight of 2KN by means of two rope fixed to the weight. One rope is inclined at 45° & other rope at 30° with their vertical. Find the tension in each rope.

Sol^n;





Given,

Weight of the body, $W = 2KN$.

Let, T_1 & $T_2 \longrightarrow$ The tension in the two ropes.

Acc. to Lami's Theorem, from the figure, we have,

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{2}{\sin 75^\circ}$$

Either

$$T_1 = \frac{2 \sin 150^\circ}{\sin 150^\circ}$$

$$= 1.03KN$$

Or

$$T_2 = \frac{2 \sin 135^\circ}{\sin 75^\circ}$$

$$= 1.4KN$$

Type:- ~~Resolut~~ Parallelogram Numerical

18. The resultant of two forces when they act at an angle 60° is $14N$, if the same forces are acting at right-angle then their resultant is $\sqrt{136}N$. Determine the mag. of the two forces.

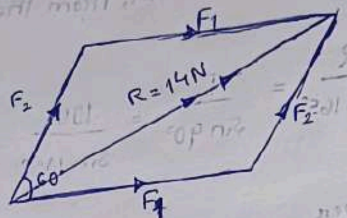


Fig.:-1

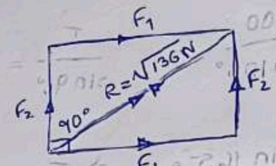


Fig:-2

Sol^n,

Let,

F_1 & F_2 = The forces whose mag. is reqd to be found out.

From Fig.1,

Acc. to llgm law of forces, we know that.

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$14 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ}$$

$$(14)^2 = F_1^2 + F_2^2 + \cancel{8}F_1F_2$$

~~Putting equations~~

$$196 = F_1^2 + F_2^2 + F_1F_2 \longrightarrow (i)$$

From fig 2

Acc. to llgm law of forces.

WKT,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\sqrt{136} = \sqrt{F_1^2 + F_2^2 \cancel{\quad}}$$

$$136 = F_1^2 + F_2^2 \cancel{\quad} \longrightarrow (ii)$$

Putting the value of eq^n (ii) in eq^n (i), we have,

$$F_1^2 + F_2^2 + F_1F_2 = 196$$

$$136 + F_1F_2 = 196$$

$$F_1F_2 = 196 - 136$$

$$F_1F_2 = 60 \longrightarrow (iii)$$

We may use that,

$$(F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1F_2$$

$$\Rightarrow (F_1 + F_2)^2 = 136 + 2\times 60$$

$$\Rightarrow (F_1 + F_2)^2 = 256$$

$$\Rightarrow F_1 + F_2 = \sqrt{256}$$

$$\therefore F_1 + F_2 = 16 \longrightarrow (iv)$$

$$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2$$

$$(F_1 - F_2)^2 = 136 - 2\times 60$$

$$(F_1 - F_2)^2 = 16$$

$$(F_1 - F_2) = \sqrt{16}$$

$$F_1 - F_2 = \cancel{16}\ 4 \longrightarrow (v)$$

Eq^n (iv) + (v) $\Rightarrow 2F_1 = 20$

$\Rightarrow F_1 = 10N$

Putting $F_1 = 10N$ in eq^n (iv)

$10 + F_2 = 16 \Rightarrow F_2 = 6N$

19. The resultant of two forces when they act at an angle of 60° is $\sqrt{13}N$. If the same forces are acting at right angle then their resultant is $\sqrt{10}N$. Determine the mag. of two forces.

Sol^n,

Let,

F_1 & F_2 = The forces whose mag. is to be found out.

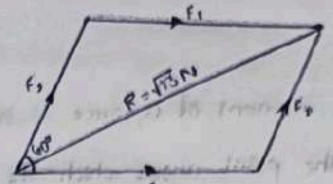


Fig. 1

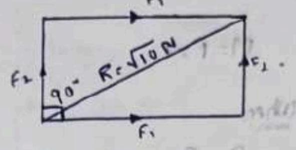


Fig. 2

From figure. 1

Acc. to parallelogram law of forces, WKT,

$$R=\sqrt{(F_1)^2+(F_2)^2+2F_1F_2\cos\theta}$$

$$\sqrt{13}=\sqrt{F_1^2+F_2^2+F_1F_2}$$

$$(\sqrt{13})^2=F_1^2+F_2^2+F_1F_2$$

$$13=F_1^2+F_2^2+F_1F_2 \;----\rightarrow(i)$$

From figure. 2 :-

Acc. to parallelogram law of forces, WKT,

$$R=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$$

$$\sqrt{10}=\sqrt{F_1^2+F_2^2+2F_1F_2}$$

$$10=F_1^2+F_2^2 \;----\rightarrow(ii)$$

Putting the value of eq^n (ii) in (i)

$$13=10+F_1F_2$$

$$F_1F_2=3 \;----\rightarrow(iii)$$

We Know that,

$$(F_1+F_2)^2=\{F_1^2+F_2^2+2F_1F_2\}$$

$$\Rightarrow (F_1+F_2)^2=10+6$$

$$\Rightarrow (F_1+F_2)=\sqrt{16}$$

$$\Rightarrow F_1+F_2=4 \;----\rightarrow(iv)$$

$$(F_1-F_2)^2=F_1^2+F_2^2-2F_1F_2$$

$$(F_1-F_2)^2=10-6$$

$$(F_1-F_2)^2=4$$

$$(F_1-F_2)=\sqrt{4}$$

$$F_1-F_2=2 \;----\rightarrow(v)$$

Eq^n (iv) + (v)

$$\Rightarrow F_1+F_2=4$$
$$F_1-F_2=2$$
$$2F_2=2$$
$$F_2=1N$$

Putting $F_2=1N$ in eq^n (iv),

$$\Rightarrow F_1+1=4$$
$$\Rightarrow F_1=3N$$

$\therefore F_1=3N,\ F_2=1N$

Date - 13.04.2025

Chapter:-2

* **Define Moment.** {2007, 2008, 2009, 2011, 2012(B), 2014, 2015(B), 2018}

→ Moment is the turning effect produce by a force on the body on which it act.

OR

The moment of a force is equal to the product of the force and the perpendicular distance of the point about which the moment is required and the line of action of the force.

Mathematically,

$$M = P \times L.$$

When

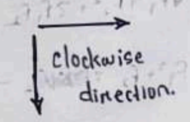
P = Force acting on the body.

L = Perpendicular distance.

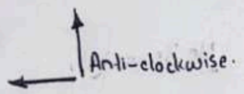
Unit of moment: If unit of force is given by N and distance is given by meter. Then unit of moment Nm.

* **Types of Moment:-**

- Clockwise moment:- It is the moment of force whose effect is to rotate the body about the point in the same direction in which the hand of Clockbow.



- Anti-clockwise moment:- It is the moment of a force whose effect is to rotated the body about the point in the opposite direction in which the hand of clockbow.



- General Sign Convention:-

  - General clockwise moments considered as '+'.
  - " anti-clockwise " " " '−'.

* **Low of Moment/or Varignon's Theorem:-** {2006, 2007, 2008, 2009, 2011, 2012 (B), 2014, 2015, 2016}.

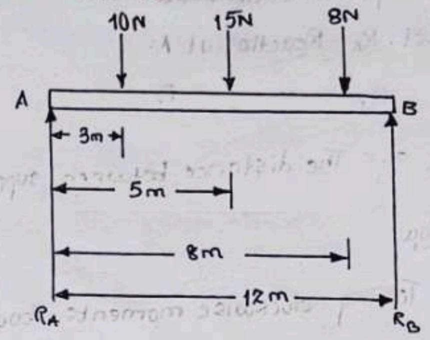
If a no. of co-planar forces are acting simultaneously on a particle that the algebraic sum of moment of all the force about any point is equal to the moment of their resultant force about the same point.

The algebraic sum of all the clockwise moment is equal to the algebraic sum of all the anticlockwise moment.

Numericals:

Q1. A Horizontal beam of 12m long is simply supported at the end and carry 10N, 15N, 8N load at a distance of 3m, 5m, 8m from the left hand and, find the reactions at the supports.
2007(B), 2008, 2008(B), 2009, 2011, 2012(B), 2013, 2015, 2016, 2017, ~~2018~~, 2019.}

Soln,



Given,

Length of the beam, Span = 12m.

Let,

R_A = Reaction at A.

R_B = Reaction at B.

Now,

Taking clockwise moments about A, we have.

$(10\times3)+(15\times5)+(8\times8)$

$=30+75+64$

$=169$ Nm.

Again

Taking anti-clockwise moment about A, we have,

$(R_A\times0)+(R_B\times12)$

$\Rightarrow 12R_B$ Nm

According to Question,

WKT,

Acc: to law of moment,

Sum of clockwise moments = Sum of anti-clockwise moments.

$\Rightarrow 169 = 12R_B$

$\Rightarrow R_B = \frac{169}{12}$

$\Rightarrow R_B = 14$N

From the figure, we have,

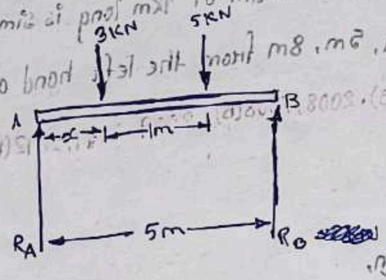
$R_A + R_B = 10N+15N+8N$

$R_A = 19N$

Date:- 17.01.2025

2. A beam AB of length 5m supported at A and B carry 2 point load W_1 & W_2 of 3KN & 5KN which are 1m apart. If the reaction at B is 2KN more than that of A, find the distance between the support A. & load 3KN. {2014, 2017(B), 2021}.

Sol^n



Given,

Length of Beam/Span = 5m

Let, R_A = Reaction at A.

$R_A = 2 + R_B$

Let x = The distance between support A & load 3KN.

Now,

Taking clockwise moments about A, we have.

$\Rightarrow (3 \times x) + [5 \times (x+1)]$ Nm.

$\Rightarrow 3x + 5x + 5$

$\Rightarrow 8x + 5$ KNm

Again,

Taking anti-clockwise moments about A, we have,

$\Rightarrow (R_A \times 0) + (R_B \times 5)$

$\Rightarrow 5R_B$ KNm.

Acc. to law of moments, WKT,

Sum of the clockwise moments = Sum of the anti-clockwise moments.

$\Rightarrow 8x + 5 = 5R_B \longrightarrow (1)$

$\Rightarrow$ From the fig we have,

$\Rightarrow W_1 + W_2 = R_A + R_B$

$\Rightarrow 3KN + 5KN = R_B - 2 + R_B$

$\Rightarrow 10 = 2R_B$

$\Rightarrow R_B = 5$ KN.

By the given condition, we have

$R_B = 2 + R_A$

$R_A = R_B - 2$.

Now,

Putting the value of R_B in eq^n no. (1)

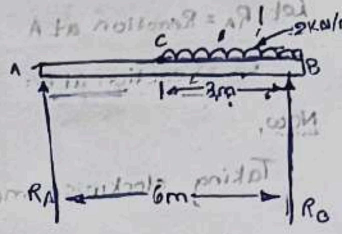
$8x + 5 = 5R_B \Rightarrow 8x = 20 \Rightarrow x = 2.5$m

$8x + 5 = 25 \Rightarrow x = 20/8$

03. A simply supported beam of 6m span is carrying a uniformly distributed load {UDL} of 2KN/m over a length of 3m from the right end. Calculate the reactions at the supports.

{2008, 2013(B), 2015, 2019}.

Sol^n.



Given,

Length of Beam or Span = 6m.

Let,

R_A = Reaction at A.

R_B = " " B.

Now,

Taking clockwise moments about A, we have,

$$\left[\underset{F}{(2\times3)}\times\underset{d}{4.5}\right] \text{ KNm}$$

$\Rightarrow$ 27 KNm

And,

Taking anti-clockwise moments about A, we have,

$$(R_A\times0)+(R_B\times6)\text{ KNm}$$

$\Rightarrow$ $6R_B$ KNm.

Acc. to law of moments, WKT,

Sum of the clockwise moments = Sum of the anti-clockwise moments

$\Rightarrow$ $27 = 6R_B$

$\Rightarrow$ $R_B = \frac{27}{6}$

$\Rightarrow$ R_B = 4.5KN

From fig, we have,

$\Rightarrow$ $R_A+R_B = 2\times3$

$\Rightarrow$ $R_A+4.5 = 6$

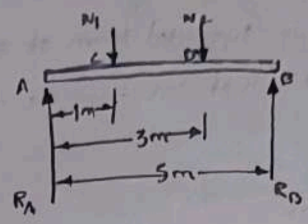
$\Rightarrow$ R_A = 1.5KN

Date:- 20.04.2025

04. A beam 5m long is simply supported at the end A & B. Two point load W_1 & W_2 are placed at C&D at a distance of 1m & 3m respectively from the end A. If the reaction at A is twice the reaction at B, find the ratio of load W_1 & W_2.

P.T.O

Given,



Length of Beam or Span = 5m

Let, R_A = Reaction at A.

R_B = Reaction at B.

$R_A = 2R_B$

Now,

Taking clockwise moments about A, we have

$(W_1 \times 1) + (W_2 \times 3)$.

$\Rightarrow W_1 + 3W_2$

Again,

Taking anti-clockwise moments about A, we have

$(R_A \times 0) + (R_B \times 5)$

$\Rightarrow 5R_B$.

Acc. to law of moments, WKT,

Sum of the clockwise moments = Sum of the anti-clockwise moments.

$\Rightarrow (W_1) + 3W_2 = 5R_B \rightarrow (i)$

From the figure, we have

$\Rightarrow W_1 + W_2 = R_A + R_B$ | By the given condition, we have $R_A = 2R_B$

$\Rightarrow W_1 + W_2 = 2R_B + R_B$

$\Rightarrow W_1 + W_2 = 3R_B \rightarrow (ii)$

Eqⁿ (i) × 3 − Eqⁿ (ii) × 5

$\Rightarrow 3W_1 + 9W_2 = \cancel{15R_B}$

$5W_1 + 5W_2 = \cancel{15R_B}$

$(-)\quad(-)$

$-2W_1 + 4W_2 = 0$

$\Rightarrow 4W_2 = 2W_1$

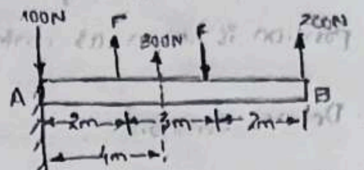
$\Rightarrow \frac{4}{2} = \frac{W_1}{W_2}$

$\frac{4}{2} = \frac{W_1}{W_2}$

$\Rightarrow \frac{W_1}{W_2} = \frac{2}{1}$

$\therefore W_1 : W_2 = 2:1$

5. Find the value of force P and F so that the four forces shown in the figure, produce an upward resultant of 300N acting at 4m from the left ~~hand~~ end A of the bar



Given,

Length of the Beam = 7m

Let,

~~R_A = Reaction at A.~~

~~R_B = Reaction at B.~~ The forces whose mag. are required to be found out.

Now,

Taking clockwise moment about A, we have

$\Rightarrow (100\times0) + (F\times5)$ Nm

$\Rightarrow 5F$ Nm

and,

Taking anti-clockwise moments about A, we have

$\Rightarrow (P\times2) + (300\times4) + (200\times7)$

$\Rightarrow 2P + 2600$ Nm.

Acc. to law of moment, we know that,

Sum of the clockwise moments = Sum of the anti-clockwise moments.

$5F = 2P + 2600$.

$\Rightarrow 5F - 2P = 2600$ ---→ (i)

From the figure we have,

$\Rightarrow 100 + F = P + 300 + 200$

$\Rightarrow 100 + F = P + 500$

$\Rightarrow F - P = 400$ ——→ (ii)

Eqn (i) × 1 − (ii) × 5

$\Rightarrow 5F - 2P = 2600$

$5F - 5P = 2000$

(−) (+) (−)

$-3P = 600$

$P = 200N$

∴ Putting P in eqn (i)

$5F - 1200 = 2600$ ←— Wrong

$\Rightarrow 5F = 1400$

∴ Putting P = 200N in eqn (ii)

~~$5F - 2P = 2600$~~

~~$\Rightarrow 5F - 400 = 2600$~~

~~$\Rightarrow 5F = 2200$~~

~~$\Rightarrow F$~~

$F - P = 400$

$F - 200 = 400$

$F = 400 + 200$

$F = 600N$.

∴ $F = 600N$, $P = 200N$.

Q. Define Centre of Gravity (C.G.)

→ It is a point through which the whole weight of the body act, irrespect of its position is known as centre of Gravity.

Q. Define Centroid.

→ The plane figure like triangle, square, pentagon, hexagon, circle, etc. Have only area but no mass, the centre of area of such figure is known as Centroid.

Important formula:-

Let,

$\bar{X}$ = The C.G. w.r.t. some axis of reference.

$\bar{Y}$ = The C.G. w.r.t. some axis of reference.

and,

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots}$$

where,

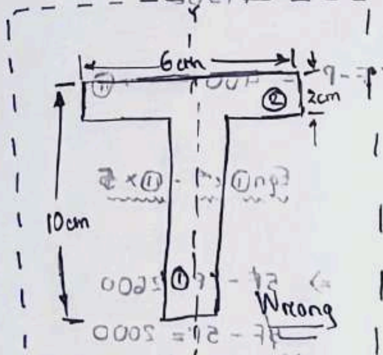
a_1, a_2, a_3 are the area of different element.

x_1, x_2, x_3 are the distance on XX axis.

y_1, y_2, y_3 are the distance on YY axis.

Numerical:-

1. Find the C.G. of a 10cm × 6cm × 2cm T-section.



Sol^n,

As the section is symmetrical about YY axis.

Therefore, C.G. will lie on the axis.

Split up the whole section into two rectangles — ①, ⑪

Let, Bottom face be the axis of reference.

From, rectangle ① we have,

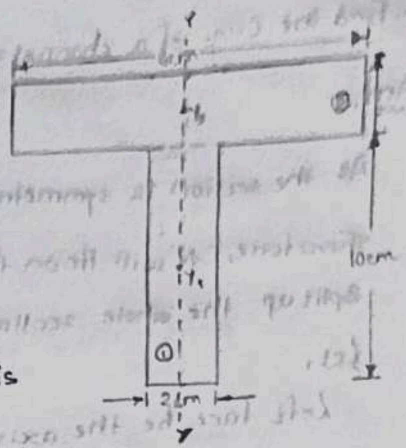
$a_1 = 8 \times 2 = 16cm^2$

$y_1 = \frac{8}{2} = 4cm$

From rectangle ② we have,

$a_2 = 6\times2 = 12cm^2$

$y_2 = 8 + \frac{2}{2} = 9cm$



We know that,

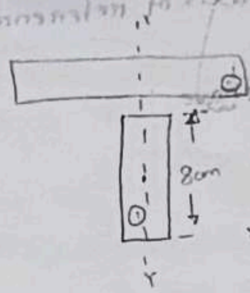
The distance between C.G. of the section and the reference is

$$\Rightarrow \bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2}$$

$$= \frac{16\times4 + 12\times9}{16+12}$$

$$= \frac{64+108}{28}$$

$$= 6.14\ cm$$

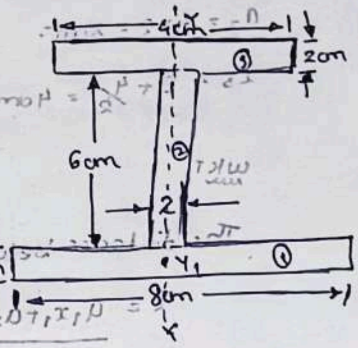


Date:- 24.04.2025

02. An 'I' section has the following dimensions:-

- Bottom Flange = 8cm×2cm.
- Web = 6cm×2cm
- Top Flange = 4cm×2cm.

Determine the C.G. of the section.



Sol^n.

As the section is symmetrical about Y-Y axis

Therefore, C.G. will lie on the axis.

Split up the whole section into three rectangles — ①, ②, ③

Let,

Bottom face be the axis of reference.

From rectangle ① we have

$a_1 = 8\times2$

$= 16cm^2$.

$y_1 = \frac{2}{2} = 1cm$.

From rectangle ②, we have

$a_2 = 6\times2 = 12cm^2$.

$y_2 = 2 + \frac{6}{2} = 5cm$.

From rectangle ③ we have

$a_3 = 4\times2 = 8cm^2$.

$y_3 = 8 + \frac{2}{2} = 9cm$.

WKT,

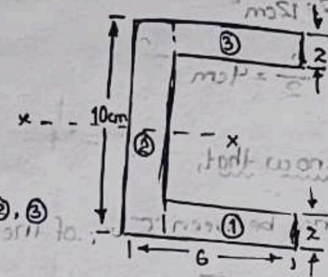
The distance between C.G. of the section and the reference is

$$\Rightarrow \bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1+a_2+a_3}$$

$$= \frac{16\times1 + 12\times5 + 8\times9}{16+12+8} = 4.1cm$$

Q3. Find the C.G. of a channel section of 10cm x 6cm x 2cm

Ans:-



As the section is symmetrical along x-x axis

Therefore, C.G. will lie on the axis.

Split up the whole section into three rectangle- ①, ②, ③

Let,

Left face be the axis of reference.

From rectangle ①, we have

$a_1 = 4 \times 2 = 8 cm^2$.

$x_1 = 2 + 4/2 = 4 cm$

From rectangle ②, we have

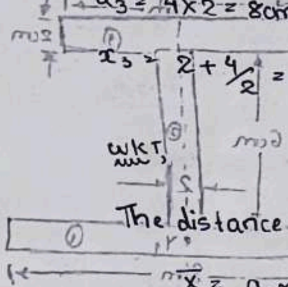
$a_2 = 10 \times 2 = 20 cm^2$

$x_2 = 2/2 = 1 cm$.

From rectangle ③, we have,

$a_3 = 4 \times 2 = 8 cm^2$.

$x_3 = 2 + 4/2 = 4 cm$



WKT,

The distance between C.G. of the section and the reference is

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

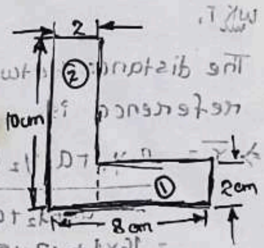
$$= \frac{8 \times 4 + 20 \times 1 + 8 \times 4}{8 + 20 + 8}$$

$$= \frac{32 + 20 + 32}{36}$$

$$= \frac{84}{36}$$

$$= 2.33 cm$$

Q4. Find the C.G. of an angle section for the dimension 10cm x 8cm x 2cm.



Solⁿ:

As the section is unsymmetrical about both the axis

Therefore, C.G. will lie

Split up the whole section into two rectangles — ① & ⓘⓘ.

Let,

Bottom face and left face be the axis of references.

From rectangle ①, we have,

$a_1 = 6 \times 2 = 12 cm^2$.

$x_1 = 2 + 6/2 = 5 cm$

$y_1 = 2/2 = 1 cm$.

From rectangle ⓘⓘ, we have,

$a_2 = 10 \times 2 = 20 cm$.

$x_2 = 2/2 = 1 cm$

$y_2 = 10/2 = 5 cm$.

WKT,

The distance between C.G. of the section and the references are:-

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} \qquad \bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{12 \times 5 + 20 \times 1}{12 + 20} \qquad = \frac{12 \times 1 + 20 \times 5}{12 + 20}$$

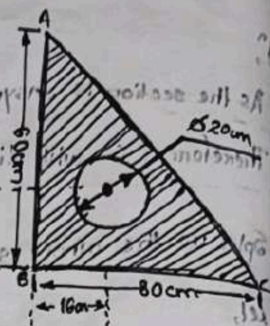
$$= \frac{60 + 20}{32} \qquad = \frac{12 + 100}{32}$$

$$\bar{x} = 2.5 cm \qquad \bar{y} = 3.5 cm.$$

Q5. A triangular sheet ABC is of uniform thickness and two side AB & BC are 60cm & 80cm long. The angle ABC is 90°. A hole of 20cm diameter is punched with a centre of 16 cm from AB & 24cm from BC. Determine the C.G. of the punched sheet.

As the section is unsymmetrical about both the axis.

Split up the whole section



Let,

Bottom face and left face be the axis of references.

From The Triangle, we have:-

$a_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 80 \times 60 = 2400 cm^2$.

$x_1 = b/3 = 80/3 = 26.66$

$y_1 = h/3 = 60/3 = 20cm$

From the circle, we have,

$a_2 = \pi r^2 = 3.14 \times 10^2 = 314 cm^2$.

$x_2 = 16cm$

$y_2 = 24cm$.

WKT,

The distance between the C.G. of the sheet and the references are,

$$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} \qquad \bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{2400 \times 26.66 + 314 \times 16}{2400 + 314} \qquad = \frac{2400 \times 20 + 314 \times 24}{2400 + 24}$$

$$= \frac{63984 + 5024}{2714} \qquad = \frac{48000 + 7536}{2424}$$

$= 25.42 cm.$ $\qquad = 22.91$

Moment of Inertia

*** Define M.I.**

→ Moment is the product of the force and perpendicular distance. If this moment is again multiplied by the perpendicular distance between the point & the line of action of the force then this quantity is called Moment of Inertia.

*** M.I. of the Plane Area:-**

$$M.I.(I) = a_1 r_1^2 + a_2 r_2^2 + \ldots\ldots$$

where,

a_1, a_2, a_3 are the area of different elements.

r_1, r_2, r_3 are the corresponding distance of the element from the line about which the M.I. is required to be found out.

Unit of M.I.

If unit of area is given in m^2 and unit of distance is given in m, then unit of M.I. $= m^4$.

{Important}

*** Theorem of Parallel Axis:-** {2006, 2008, 2010, 2011, 2013(B), 2015, 2015(B), 2016, 2018(B), 2020}.

If the M.I. of a plane area about the axis through the C.G. is denoted by (I_G), then the M.I. of the area about any other parallel axis AB, parallel to the 1st and at a distance 'h' from the C.G. is given by $\boxed{I_{AB} = I_G + a.h^2}$

where,

I_{AB} = M.I. of the area about parallel axis AB.

I_G = M.I. of the area through C.G.

a = Area of the section.

h = Distance between C.G. of the section and the axis AB.

Consider a strip on the circle.

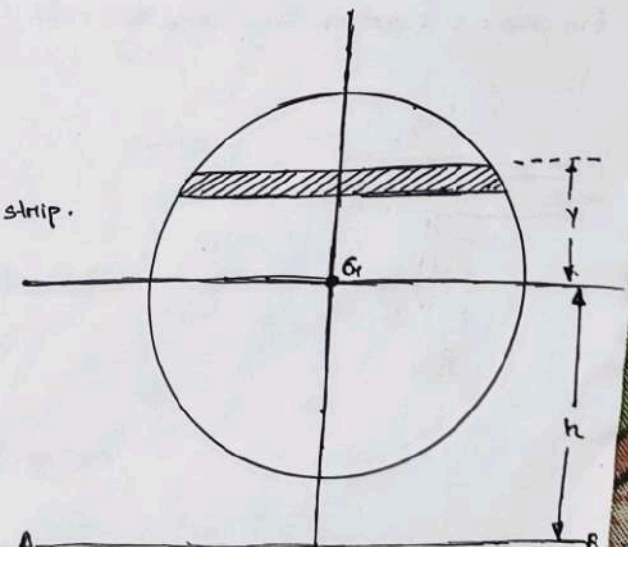
Let,

y → distance between C.G. of the section & the strip.

h → distance between C.G. of the section & the parallel axis AB.

a → Area of the section.

δ_a → Area of the strip.



WRT,

M.I. of the strip about x–x axis is

$$I = ah^2$$

$$I_{xx} = \delta a \times y^2$$

M.I. of the section through C.G. is

$$I_G = \Sigma \delta_a \times y^2$$

M.I. of the section about parallel axis AB is

$$I_{AB} = \Sigma \delta_a \times (y+h)^2$$

$$= \Sigma \delta a \times (y^2 + h^2 + 2yh)$$

$$= \Sigma \delta a y^2 + \Sigma \delta_a \times h^2 + \Sigma \delta a \times 2yh$$

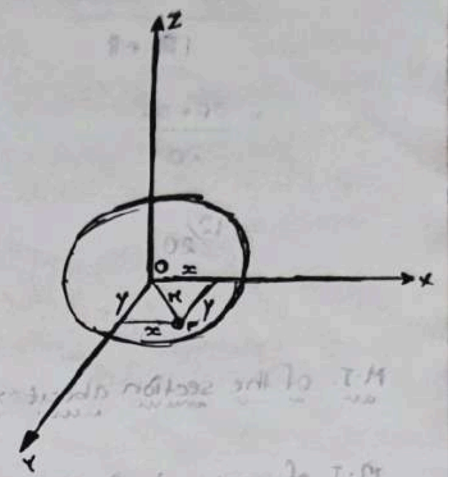
$$= I_G + ah^2 + 0$$

$$\boxed{I_{AB} = I_G + ah^2}$$

Hence Proved.

* Theorem of Perpendicular Axis :- (Important)

If I_{XX} and I_{YY} be the M.I. of a plane section about two perpendicular axis meeting at O, then the M.I. I_{ZZ} about the axis ZZ perpendicular to the plane & passing through the intersection of XX and YY is given by $\boxed{I_{ZZ} = I_{XX} + I_{YY}}$



Consider a lamina (P) on the circle.

Let,

a → Area of the section.

δ_a → Area of the lamina.

From the fig, by the geometry, we have,

$r^2 = x^2 + y^2$ {By Pythagoras Theorem}.

WKT,

M.I. of the lamina about X–X axis is,

$$I_{XX} = \delta_a \times y^2$$

M.I. of the lamina about Y–Y axis is,

$$I_{YY} = \delta_a + x^2$$

M.I. of the lamina about Z–Z axis is,

$$I_{ZZ} = \delta_a \times r^2$$

$$= \delta_a \times (x^2 + y^2)$$

$$= \delta_a \times x^2 + \delta_a + y^2$$

$$\boxed{\therefore I_{ZZ} = I_{XX} + I_{YY}}$$

Hence Proved.

Date :- 15.05.2025

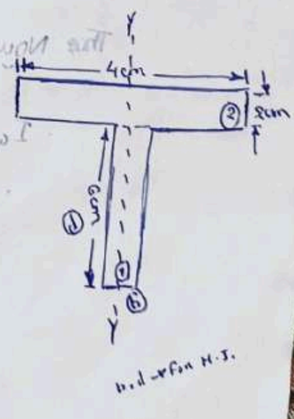
* Find the M.I. of a T-Section with top flange = 4cm × 2cm, Web = 6cm × 2cm about X–X axis and Y–Y axis passing through the C.G. of the section.

Soln.

As the section is symmetrical about Y–Y axis.

Therefore, C.G. will lie on the axis.

Split up the whole section into two rectangles — ① and ②.



Let,

Bottom face be the axis of reference.

h.d → for M.I.

From rectangle ①, we have

$a_1 = 6 \times 2 = 12cm^2$

$y_1 = 6/2 = 3cm$

From rectangle ②, we have,

$a_2 = 4 \times 2 = 8cm^2$

$y_2 = 6 + 2/2 = 7cm.$

Ans,

The distance between C.G. of the section and the reference is.

$$\bar{Y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{12 \times 3 + 8 \times 7}{12 + 8}$$

$$= \frac{36 + 56}{20}$$

$$= 92/20$$

$$= 4.6 \text{ cm}$$

M.I. of the section about X–X axis.

M.I. of rectangle ① through C.G. is,

$$I_{G_1} = \frac{bd^3}{12} = \frac{2 \times 6^3}{12}$$

$$= 36 \text{cm}^4$$

The distance between C.G. of rectangle ① and X–X axis is,

$$h_1 = \bar{Y} - y_1$$

$$= 4.6 - 3$$

$$= 1.6 \text{cm}$$

Therefore, M.I. of rectangle ① about X–X axis is

$$I_1 = I_{G_1} + a_1 h_1^2$$

$$= 36 + 12 \times (1.61)^2$$

$$= 66.7 \text{cm}^4$$

~~The~~ Now, M.I. of rectangle ② ~~about~~ through C.G. is

$$I_{G_2} = \frac{bd^3}{12}$$

$$= \frac{4 \times 2^3}{12}$$

$$= 2.6 \text{cm}^4$$

The distance between C.G. of rectangle ② and x-x axis is,

$h_2 = y_2 - \bar{Y}$

$= 7 - 4.6$

$= 2.4\,cm$

Therefore, M.I. of rectangle ② about x-x axis,

$I_2 = I_{G_2} + a_2 h_2^2$

$= 2.6 + 8 \times (2.4)^2$

$= 48.68\,cm^4$

Therefore, finally M.I. of the whole section about x-x axis is

$I = I_1 + I_2$

$= 66.7 + 48.6$

$= 115.3\,cm^4$ Ans

M.I. of the section about Y-Y axis

M.I. of rectangle ① ~~through~~ about Y-Y axis is,

$I_1 = \frac{db^3}{12} = \frac{6 \times 2^3}{12} = 4\,cm^4$

M.I. of rectange ② about Y-Y axis is,

$I_2 = \frac{db^3}{12} = \frac{2 \times 4^3}{12} = 10.6\,cm^4$

Therefore, M.I. of the whole section about Y-Y axis is,

$\therefore I_4 = I_1 + I_2$

$I = 4 + 10.6$

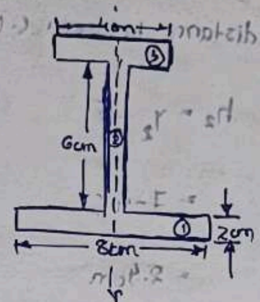
$I = 14.6\,cm^4$ Ans

Q. Find the M.I. of an I-section has the following dimensions

- Top flange = 4cm x 2cm
- Web = 6cm x 2cm.
- Bottom flange = 8cm x 2cm

about the horizontal axis passing through the C.G. of the section.

Date:- 18.05.2025



As the section is symmetrical about Y-Y axis.

Therefore, C.G. will lie on the axis

Split-up the whole section into three rectangles ①, ②, ③.

Let,

Bottom face be the axis of reference.

From rectangle ① we have,

$a_1 = 8 \times 2 = 16 cm^2$.

$Y_1 = \cancel{8\times2} \ 2/2 = 1 cm$

From rectangle ②, we have,

$a_2 = 6 \times 2 = 12 cm^2$

$Y_2 = 2 + 6/2 = 5 cm$

From rectangle ③, we have,

$a_3 = 4 \times 2 = 8 cm^2$

$Y_3 = 2 + 6 + 2/2 = 9 cm$.

WKT,

The distance between C.G. of the section and the reference is

$$\overline{Y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{16 \times 1 + 12 \times 5 + 8 \times 9}{16 + 12 + 8}$$

$= 4.1 cm$.

M.I. of the section about horizontal (x-x) axis,

M.I. of rectangle ① through C.G. is

$$I_{G_1} = \frac{bd^3}{12} = \frac{8 \times 2^3}{12} = 5.33 cm^4$$

The distance between C.G. of rectangle ① and x-x axis is

$h_1 = \overline{Y} - Y_1$

$= 4.1 - 1$

$= 3.1 cm$

Therefore, M.I. of rectangle ① through x-x axis is

$\Rightarrow I_{a_1} = I_{G_1} + a_1 h_1^2$

$= 5.33 + 16 (3.1)^2$

$= 30.94 cm^4$.

Now, M.I. of rectangle ② through C.G. is

$$I_{G_2} = \frac{bd^3}{12}$$

$$= \frac{2\times6^3}{12} = 36 cm^4$$

The distance between C.G. of rectangle ② about x-x axis is

$h_2 = y_2 - \bar{Y}$

$= 5 - 4.1$

$= 0.9 cm$

Therefore, M.I. of rectangle ② through x-x axis is

$\Rightarrow I_{a_2} = I_{G_2} + a_2 h_2^2$

$= 36 + 12\times(0.9)^2$

$= 45.72 cm^4$

Again/Similarly,

M.I. of rectangle ③ through C.G. is

$$I_{G_3} = \frac{bd^3}{12}$$

$$= \frac{4\times8}{12} = 2.6 cm^4$$

The distance between C.G of rectangle ③ about x-x axis is

$h_3 = y_3 - \bar{Y}$

$= 4.9 cm$

Therefore, M.I. of rectangle ③ through x-x axis is

$I_3 = I_{G_3} + a_3 h_3^2$

$= 2.6 + 8\times(4.9)^2 = 194.68 cm^4$

Therefore,

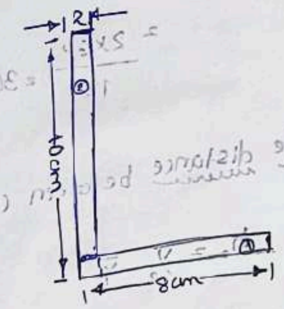
finally M.I. of the whole section about X-X axis

$I = I_1 + I_2 + I_3$

$= 30.9 + 45.7 + 194.6$

$= 271.2\ cm^4$

Q. Find the M.I. of an angle section about the X-X axis and Y-Y axis for the dimension 10cm x 8cm x 2cm.



Sol^n,

As the section is unsymmetrical about both the axis. Split-up the whole section into two rectangle - ①, ②.

Let,

Bottom face and left face be the axis of references.

From rectangle ①, we have,

$a_1 = 6 \times 2 = 12\ cm^2$

$\cancel{y_1 = 6/2 = 3\ cm}$ $\quad x_1 = \frac{6}{2} + 2 = 5$

$\cancel{x_1 = 2/2 = 1\ cm}$ $\quad y_1 = \frac{2}{2} = 1cm.$

From rectangle ② we have

$a_2 = 10 \times 2cm = 20cm^2$

$x_2 = \frac{2}{2} = 1cm$

$y_2 = \frac{10}{2} = 5cm.$

WKT,

The distance between c.g. of the section and the axis of references is

$\bar{X} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$ $\quad \bar{Y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$

$= \frac{12 \times 5 + 20 \times 1}{12 + 20}$ $\quad = \frac{12 \times 1 + 20 \times 5}{12 + 20}$

$= 2.5cm$ $\quad = 3.1cm$

M.I. of the section about X-X axis

M.I. of rectangle 1 through C.G. is

$$I_{G_1} = \frac{bd^3}{12}$$

$$= \frac{6 \times 2^3}{12}$$

$$= 4\,cm^4.$$

The distance between C.G. of rectangle 1 and X-X axis is.

$$h_1 = \bar{Y} - Y_1$$

$$= 3.1 - 1$$

$$= 2.1cm$$

Therefore, M.I. of rectangle 1 about X-X axis is.

$$I_1 = I_{G_1} + a_1h_1^2$$

$$= 4 + 12 \times (2.1)^2$$

$$= 56.92\,cm^4$$

Similarly,

M.I. of rectangle 2 through C.G. is

$$I_{G_2} = \frac{2 \times 10^3}{12} = 166.6\,cm^4.$$

The distance between C.G. and rectangle 2 and X-X axis is

$$h_2 = Y_2 - \bar{Y} = 5 - 3.1 = 1.9cm$$

Therefore, M.I of rectangle 2 about X-X axis is:-

$$I_2 = I_{G_2} + a_2h_2^2 = 166.6 + 20 \times (1.9)^2 = 238.8\,cm^4.$$

Therefore, finally M.I. of the whole section about X-X axis is.

$$I = I_1 + I_2 = 56.92 + 238.8 = 295.72\,cm^4$$

M.I. of the section about Y-Y axis

M.I. of rectangle ① through C.G. is

$$I_{G_1} = \frac{db^3}{12} = \frac{2\times6^3}{12} = 36 cm^4$$

The distance between C.G. and rectangle ① and Y-Y axis.

$$h_1 = \bar{x} - x_1 = 5-2.5 = 2.5cm.$$

Therefore, M.I. of rectangle ① about Y-Y axis is.,

$$I_1 = I_{G_1} + a_1h_1^2$$

$$= 36 + 75 = 111cm^4.$$

Similarly, M.I of rectangle ② through C.G. is.

$$I_{G_2} = \frac{db^3}{12} = \frac{10\times2^3}{12} = 26.66cm^4.$$

The distance between C.G. of rectangle ② and Y-Y axis is

$$h_2 = \bar{x} - x_2$$

$$= 2.5-1 = 1.5cm.$$

Therefore, M.I. of rectangle ② about Y-Y. axis is

$$I_2 = I_{G_2} + a_2h_2^2$$

$$= 51.66 cm^4$$

Therefore, finally, M.I. of the whole section about Y-Y axis is.

$$\therefore I = I_1 + I_2 = 162.66 cm^4.$$

Date:- 22.05.2025

Friction

* Define Friction or force of friction.

→ Friction or force of friction may be defined as the opposite force which is called into play in between the surfaces of contact of two bodies when one body moves over the surface of another body.

* Types of Friction

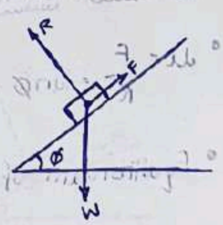
- Static Friction:- It is the friction experienced by a body when it is at rest.
- Dynamic Friction:- It is the friction experienced by a body when it is in motion. It is also called Kinetic Friction.
    - Sliding Friction:- It is the friction experienced by a body when it slide over another body.
    - Rolling Friction:- It is the friction experienced by a body when it rolls over another body.

* Limiting Friction:- Limiting friction may be defined as the max^{m} value of frictional force which comes into play when a body just begin to slide over the surface of another body.

* Angle of friction:- The angle of inclined plane at which a body just begin to slide down the plane is called the angle of friction. This is also equal to the angle which the normal reaction makes with the vertical. Mathematically,

$$\tan \phi = \frac{F}{R}$$

$$\therefore \phi = \tan^{-1}(F/R).$$



where,

W = weight of the body.

R = Normal Reaction.

F = Frictional Force.

ϕ = Angle of Friction.

* Coefficient of Friction:- It is the ratio of limiting friction to the normal reaction between the two bodies. It is denoted by 'μ'. Mathematically,

$$\mu = F/R = \tan \phi$$

where,

F = limiting friction or fictional force.

R = Normal Reaction.

ϕ = Angle of friction.

* **Law of Static Friction and Dynamic Friction :-**

- The frictional force always act in the opposite direction to that in which the body tends to move.
- The frictional force is exactly equal to the applied force.
- The frictional force is directly proportional to the normal reaction between two surfaces.
- The frictional force depends upon the roughness of the surfaces.
- The frictional force is independent to the area of contact between two bodies.
- The frictional force remains constant for moderate speed but it decreases with the increase of speed.

* **Angle of Repose :-** The maxm inclination of the plane in which a body can remain in equilibrium over the plane entirely by the assistance of friction is called the angle of repose.

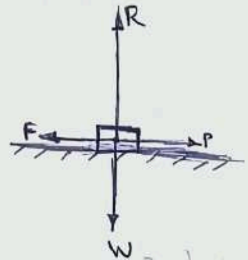
* **Value of coefficient of friction (μ) :-**
  - The value of μ in wood v/s wood = 0.4 to 0.7
  - The value of μ in wood v/s metal = 0.4 to 0.6
  - The value of μ in metal v/s metal = 0.15 to 0.25

Date :- 30.05.2025

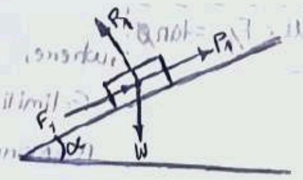
Important Formula :-

- $\mu = \frac{F}{R} = \tan\phi$
- Equilibrium of a body on a rough horizontal plane

$$\boxed{F = \mu R}$$

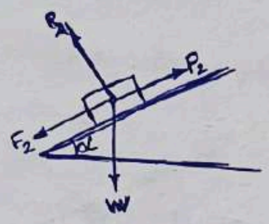


- Equilibrium of a body on a rough inclined plane subjected to a force acting along the inclined plane.
  - Minimum force (P_1) which will keep the body in equilibrium when it is at the point of sliding downwards.



$$\boxed{P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos\phi}}$$

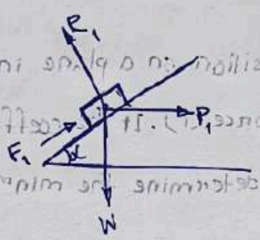
□ Maximum force (P_2) which will keep the body in equilibrium when it is at the point of sliding upwards.



$$\boxed{P_2 = W \times \frac{\sin(\alpha+\phi)}{\cos\phi}}$$

• Equilibrium of a body on a rough inclined plane subjected to a force acting horizontally.

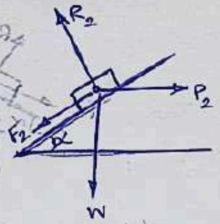
□ Minimum Force (P_1) which will keep the body in equilibrium when it is at the point of sliding downwards.



$\boxed{P_1 = W \times \tan(\alpha - \phi)}$; when $\alpha > \phi$

$\boxed{P_1 = W \times \tan(\phi - \alpha)}$; when $\phi > \alpha$

□ Maximum force (P_2) which will keep the body in equilibrium when it is at the point of sliding upwards.

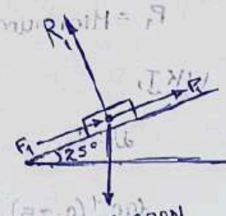


$$\boxed{P_2 = W \times \tan(\alpha + \phi)}$$

Numericals :-

Q1. A body of weight 500 N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane. Determine the min^m and max^m value of P, if the angle of friction is 20°

⇒ Min^m force (P_1)



Given

Weight of the body (W) = 500N.

Angle of Inclined plane (α) = 25°

Angle of Friction, (ϕ) = 20°

Let,

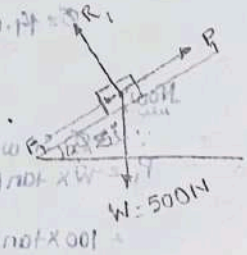
P_1 = Min^m force.

WKT,

$$P_1 = W \times \frac{\sin(25-20)}{\cos(20)}$$

$$= 500 \times \frac{\sin(5)}{\cos 20}$$

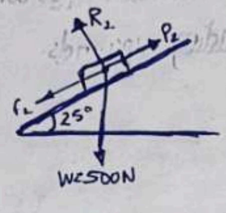
$$\Rightarrow 46.37 N$$



Force (P_2)

Let,

P_2 = Max^m Force.



WKT,

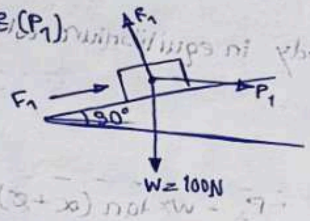
$$P_2 = W \times \frac{\sin(\alpha+\phi)}{\cos\phi}$$

$$= 500 \times \frac{\sin(25+20)}{\cos 20}$$

$$= 376.24\,N$$

Q2. An object of weight 100N is kept in position on a plane inclined 30° to the horizontal by a horizontally applied force (P). If the coeff. of friction of the surface of the inclined plane is 0.25, determine the min^m and max^m mag. of the force.

Sol^n, Minimum Force (P_1)



Given,

Weight of the body = 100N

Angle of Inclined plane, $\alpha = 30°$

Coeff. of Friction, $\mu = 0.25$

Let,

P_1 = Minimum Force.

WKT,

$\mu = \tan\phi \Rightarrow \tan\phi = \mu$

$\tan^{-1}(0.25) = \phi$

$\phi = 14.03°$

Now,

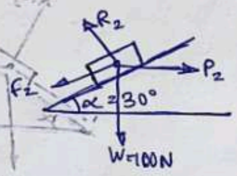
$\because \alpha > \phi$, so we may use that

$P_1 = W \times \tan(\alpha - \phi)$

$= 100 \times \tan(30-14)$

$= 28.67\,N$

Maximum Force (P_2)



Let,

P_2 = Max^m force.

WKT,

$P_2 = W \times \tan(\alpha+\phi)$

$= 100 \times \tan(30+14)$

$= 96.5\,N$

Q3. A ~~body resting on a rough~~ load of 1.5 KN resting on an inclined rough plane can be move up the plane by a force 2KN applied horizontally or by a force 1.25 KN applied parallel to the plane. Find the inclination of the plane and the coeff. of friction.

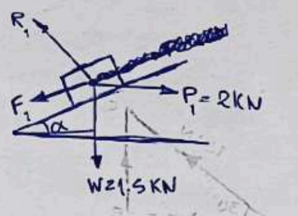


Fig.1

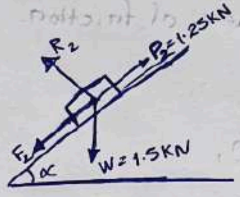


Fig.3

Weight of the body, $W = 1.5KN$

Applied force, $P_1 = 2KN$.

Applied force; $P_2 = 1.25KN$.

Let,

α = Inclination of the plane

μ = Coeff. of friction.

From Fig.1,

WKT,

$P_1 = W \times \tan(\alpha + \phi)$

$\Rightarrow 2 = 1.5 \times \tan(\alpha + \phi)$

$\Rightarrow \tan(\alpha + \phi) = \dfrac{2}{1.5}$

$\Rightarrow \alpha + \phi = \tan^{-1}(1.33)$

$\Rightarrow \alpha + \phi = 53.1^\circ \longrightarrow (1)$

From fig.2,

WKT,

$P_2 = \dfrac{W \times \sin(\alpha + \phi)}{\cos\phi}$

$\Rightarrow 1.25 = \dfrac{1.5 \times \sin(53.1)}{\cos\phi}$

$\Rightarrow \cos\phi = \dfrac{1.5}{1.25} \times \sin 53.1^\circ$

$\Rightarrow \phi = \cos^{-1}\left(\dfrac{1.5}{1.25} \times \sin 53.1^\circ\right)$

$\Rightarrow \phi = 16.3^\circ$

Now,

Putting the value of ϕ in eqn (1) we have,

$\alpha + 16.3 = 53.1^\circ$

$\Rightarrow \alpha = 36.8^\circ$

Again,

WKT,

$\mu = \tan\phi$

$\mu = \tan(16.3^\circ)$

$\mu = 0.292$

Q1. A body resting on a rough horizontal plane required a pull of 180N inclined 30° to the plane just to move it. It was found that a push of 220N inclined 30° to the plane just move the body. Determine the wt. of the body, and the coefficient of friction.

Soln,

Given,

Pull force, $P_1 = 180N$

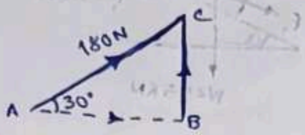
Push force, $P_2 = 220N$

$\alpha_1 = 30°$

$\alpha_2 = 30°$

$W = ?$

$\mu = ?$



$\cos 30° = \frac{AB}{AC}$

$= \frac{AB}{180}$

$AB = 180 \cos 30°$

$= 155.9N$

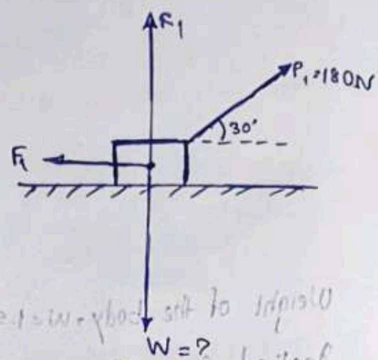
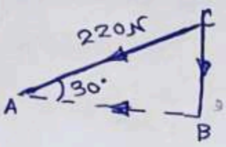


Fig:①

Let,

W = weight of the body

μ = Coeff. of Friction.



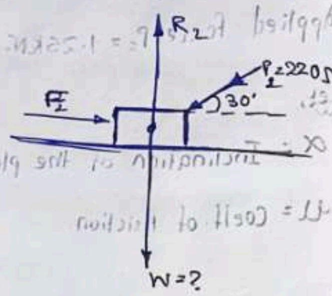


Fig:(ii)

From Fig ①.

Resolving all the forces horizontally,

$F_1 = 180\cos 30° = 155.9N.$

Resolving all the forces vertically,

$R_1 + 180\sin 30° = W.$

$R_1 + 90 = W$

$R_1 = (W-90)N$

WKT,

$F_1 = \mu R_1$

$155.9 = \mu(W-90) \text{ ---} \rightarrow (i)$

From fig ②

Resolving all the forces horizontally,

$F_2 = 220\cos 30°$

$= 190.5N$

Resolving the forces vertically,

$R_2 = W + 220\sin 30°$

$R_2 = (W+110)N$ ---(ii)

WKT

$F_2 = \mu R_2$

$190.5 = \mu(W+110) \text{ ---} \rightarrow (ii)$

Eqn (ii) - Eqn (i)

$190.5 = W\mu + 110\mu$

$\underline{{}_{(-)}155.9 = {}_{(-)}W\mu \ {}_{(+)}90\mu}$

Eq^n (ii) $\div$ (i)

$$\frac{190.5}{155.9} = \frac{\mu(W+110)}{\mu(W-90)}$$

$\Rightarrow 190.5W - 17145 = 155.9W + \cancel{14031}\ 17149$

$\Rightarrow 190.5W - 155.9W = 17149 + 17145$

$\Rightarrow 34.6W = 34294 \Rightarrow W = 991.156$

Now putting the value of W in eq^n ①

$155.9 = \mu(W-90)$

$155.9 = \mu(991.756 - 90)$

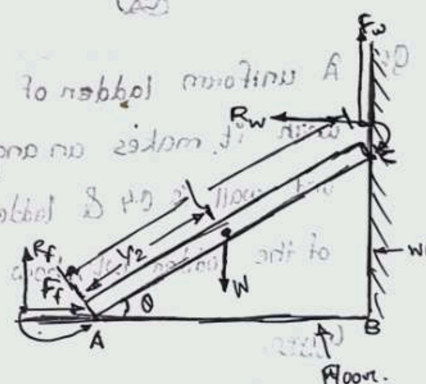
$155.9 = \mu(901.156)$

$\mu = 0.173$ Ans

* Ladder Friction:- Ladder is a device used for climbing on the wall or roof.

Q5. A uniform ladder is in equilibrium with one end resting on the ground and the other end resting against a vertical wall. If the wall and ground be both rough, the coeff. of friction, are 0.5 for all surfaces, and if the ladder be on the pt. of slipping at both ends, find the inclination of ladder without slipping.

Soln:- Given,



Coeff. of friction between ladder & Floor, $\mu_f = 0.5$

" " " " ladder & Wall, $\mu_w = 0.5$

Let,

$W \to$ Wt. of the ladder.

$l \to$ length of the ladder

$\theta \to$ Inclination of the ladder with the floor without slipping.

Resolving all the forces horizontally,

$R_w = F_f$

$R_w = \mu_f R_f$ $\quad [\because F_f = \mu_f R_f]$

$R_w = 0.5 R_f \longrightarrow (1)$

Resolving the forces vertically,

$R_f + F_w = W$

$\Rightarrow R_f + \mu_w R_w = W$

$\Rightarrow R_f + 0.5 R_w$ $(0.5 R_f) = W$

$\Rightarrow R_f + 0.25 R_f = W$

$\Rightarrow 1.25 R_f = W \Rightarrow 1.25 \times \frac{R_w}{0.5} = W$

$R_W = \frac{0.5W}{1.25}$

$\Rightarrow R_W = 0.4W$ ---- →(ii)

Now taking moments about A, we have,

Taking Clockwise Moments = Anti-clockwise Moments,

$(F_f \times 0) + (W \times \frac{l}{2}\cos\theta) = (R_f \times 0) + (R_W \times l\sin\theta) + (F_W \times l\cos\theta)$.

$\Rightarrow 0 + 0.5\,Wl\cos\theta = 0 + 0.4W\,l\sin\theta + (0.5 \times 0.4W\,l\cos\theta)$

$\Rightarrow 0.5\,Wl\cos\theta = 0.4\,Wl\sin\theta + 0.2\,Wl\cos\theta$.

$\Rightarrow 0.5\cos\theta - 0.2\cos\theta = 0.4\sin\theta$

$\Rightarrow 0.3\cos\theta = 0.4\sin\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{0.3}{0.4}$

$\Rightarrow \tan\theta = \frac{0.3}{0.4}$

$\Rightarrow \theta = 36.87°$

∴ The inclination of the ladder with the floor without slipping is 36.87°.

Q6. A uniform ladder of 4m length, rest against a vertical wall with which it makes an angle of 45°. The coeff. of friction between the ladder and wall, is 0.4 & ladder & floor is 0.5. If a man whose weight is half ($\frac{1}{2}$) of the ladder wt., how much height will it be when the ladder slip.

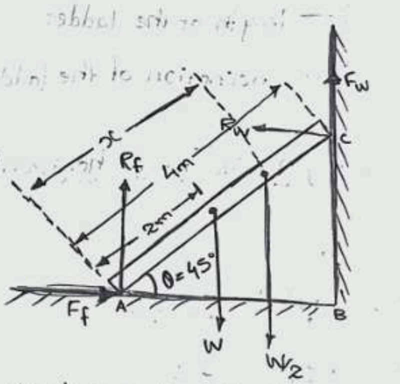
Given,

Length of ladder, $l = 4m$

Coeff. of friction between ladder & wall, $\mu_w = 0.4$

" " " " ladder " floor, $\mu_f = 0.5$

Angle between ladder & floor = 45°.



Let,

W = wt. of the ladder.

x = Distance between floor and the man standing on the ladder.

Resolving all the forces horizontally:-

$R_W = F_f$

$\Rightarrow R_W = \mu_f R_f$

$\Rightarrow R_W = 0.5 R_f \dashrightarrow (I)$

Resolving all the forces vertically,

$\Rightarrow R_f + F_w = W + \frac{W}{2}$

$\Rightarrow R_f + \mu_w R_w = 1.5W$

$\Rightarrow R_f + 0.4 \times 0.5 R_f = 1.5W$

$\Rightarrow R_f + 0.2 R_f = 1.5W$

$\Rightarrow 1.2 R_f = 1.5W$

$\Rightarrow 1.2 \times \frac{R_w}{0.5} = 1.5W$

$\Rightarrow R_w = \frac{0.5 \times 1.5W}{1.2}$

$\Rightarrow R_w = 0.625W \dashrightarrow (II)$

Now,

Taking moments about A, we have,

$(F_f \times 0) + (W \times 2\cos 45^\circ) + (\frac{W}{2} \times x \cos 45^\circ) = (R_f \times 0) + (R_w \times 4\sin 45^\circ) + (F_w \times 4\cos 45^\circ)$

$\Rightarrow 2W\cos 45^\circ + 0.5W x\cos 45^\circ = 0.625W \times 4\sin 45^\circ + 0.4 \times 0.625W \times 4\cos 45^\circ$

$\Rightarrow 2\cos 45^\circ + 0.5x\cos 45^\circ = 0.625 \times 4\sin 45^\circ + 0.4 \times 0.625 \times 4\cos 45^\circ$

$\Rightarrow 1.41 + 0.35x = 2.47$

$\Rightarrow 0.35x = 2.47 - 1.41$

$\Rightarrow x = \frac{1.06}{0.35} = 3.02 \approx 3m.$

$\therefore$ The man can climb on the ladder without sliping upto 3m (i.e., 75/cm.)

SIMPLE LIFTING MACHINE

* **Simple Machine:-** A simple machine may defined as a device which enable us to do some useful work at some point when an effort or force is applied to it at some other convenient point.

* **Compound Machine:-** A compound machine may be defined as a device consisting of a no.'s of simple machine which enable us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

* **Lifting Machine:-** It is a device which enable us to lift a heavy load {W} by applying a comparatively smaller effort {P}.

* **Mechanical Advantage {M.A.}:-** The M.A. is the ratio of weight lifted {W} to the effort applied {P}. It is always expressed in pure number.

Mathematically,

$$\boxed{M.A. = \frac{W}{P}}$$

* **Input of a Machine:-** The Input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort {P} & the distance move by the effort {y}.

Mathematically,

$$\text{Input of a Machine} = P \times y.$$

* **Output of a machine:-** The output of a machine is the actual W.D. by a machine. In a lifting machine, it is measured by the product of the wt. lifted {W} & the distance moved by the wt. {x}.

Mathematically,

$$\text{Output of a machine} = W \times x.$$

* **Efficiency:-** It is the ratio of output to the input of a machine. It is generally expressed as a percentage. Mathematically,

$$\text{Efficiency}, \eta = \frac{\text{output}}{\text{Input}} \times 100\%.$$

* **Ideal Machine:-** If the efficiency of a machine is 100%, i.e., if the output is equal to the input of the machine, then the machine is called ideal machine or perfect machine.

* **Velocity Ratio {V.R.}:-** The Velocity Ratio is the ratio of distance moved by the effort (y), to the distance moved by the load (x). It is always expressed in pure no.

Mathematically,

$$\boxed{V.R. = \frac{y}{x}}$$

* **Reversible Machine:-** A machine is said to be reversible if its efficiency is greater than 50%.

* **Non-Reversible or Irreversible or Self-Locking Machine:-** A machine is said to be non-reversible if its efficiency is less than 50%. It is also called Self-Locking Machine.

* **Law of a Machine:-** The law of a machine may be defined as the relationship between the effort applied and load lifted.

Mathematically,

The law of a lifting machine is given by,

$\boxed{P = mW + C}$, where, P → effort applied to lift the load.

m → a constant is called coeff. of friction

W → load lifted.

C → another constant which represent the machine friction.

* **Relation between M.A., V.R., and Efficiency of a Lifting Machine:-**

Consider a lifting machine

Let,

W = Load Lifted by the machine.

P = Effort applied to lift the load.

y = distance moved by the effort.

x = " " " " load.

WKT,

$$M.A. = \frac{W}{P}$$

$$V.R. = \frac{y}{x}$$

WKT,

Input of a machine = Effort Applied x Distance moved by the effort

$= P \times y$

Output of a machine = Wt. lifted x distance moved by the wt.

$= W \times x$

$\therefore$ Efficiency, $\eta = \dfrac{\text{Output}}{\text{Input}}$

$= \dfrac{W x}{P y}$

$= \dfrac{W}{P} \times \dfrac{x}{y}$

$= \dfrac{W}{P} \times \dfrac{1}{y/x}$

$= M.A. \times \dfrac{1}{V.R.}$

$$\boxed{\eta = \frac{M.A}{V.R}}$$

Important Formula:-

(A) Formula for Simple Lifting Machine or Weight Lifting Machine:-

* $M.A. = \dfrac{W}{P}$
* $V.R = \dfrac{y}{x}$
* Efficiency, $\eta = \dfrac{M.A}{V.R.} \times 100\%$.
* Law of Machine, $P = mW + C$
* Max^m $M.A. = \dfrac{1}{m}$
* Max^m efficiency $= \dfrac{1}{m \times V.R.}$
* Effort lost in friction, $F_{effort} = P - \dfrac{W}{V.R.}$

(B) Formula for Simple Wheel and Axle Machine:-

* $M.A. = \frac{W}{P}$.

* $V.R. = \frac{D}{d}$, where, $D \rightarrow$ diameter of wheel.
  $d \rightarrow$ diameter of axle.

* Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100\%$.

(C) Formula for differential wheel and axle machine.

* $M.A. = \frac{W}{P}$

* $V.R. = \frac{2D}{d_1 - d_2}$, where, $D \rightarrow$ diameter of wheel.
  $d_1 \rightarrow$ " " larger axle.
  $d_2 \rightarrow$ " " smaller "

* Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100\%$.

(D) Formula for Simple Screw Jack Machine:-

* $M.A. = \frac{W}{P}$.

* $V.R. = \frac{2\pi l}{p}$, where, $l \rightarrow$ length of the lever/handle.
  $p \rightarrow$ pitch of the screw thread.

* Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100\%$.

(E) Formula for Differential Screw Jack Machine:-

* $M.A. = \frac{W}{P}$.

* $V.R. = \frac{2\pi l}{p_1 - p_2}$ where, $l \rightarrow$ length of lever.
  $p_1 \rightarrow$ Pitch of larger screw.
  $p_2 \rightarrow$ " " smaller ".

* Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100\%$.

Q1. In a wt. lifting machine, an effort of 40N can lift a load of 1300N and an effort of 55N can lift a load of 1800N. Find the law of machine.

Also find the effort to run this machine at:-

- No load.
- A load of 100N.

Soln,

Given,

Effort applied, $P = 40N, \& 55N$,

load lifted, $W = 1300N \& 1300N$

For Simple Lifting Machine, WKT.

The law of machine is, $P = mW + C \longrightarrow (i)$.

Now, Putting the values of P and W in eqn (i),

$$40 = m \times 1300 + C \longrightarrow (ii)$$

$$55 = m \times 1800 + C \longrightarrow (iii).$$

Eqn (iii) − (ii) We get,

$$15 = m \times 500$$

$$500m = 15$$

$$m = \frac{\cancel{15}}{\cancel{500}} \frac{3}{100}$$

$$\therefore m = \frac{3}{100}$$

Again Putting the value of m in eqn (ii), we have

$$40 = \frac{3}{1\cancel{0}\cancel{0}} \times 130\cancel{0} - C$$

$$C = 40 - 39$$

$$\therefore C = 1$$

Again,

Putting the value of m & c in eqn (i) to get the law of machine.

$$P = \frac{3}{100} \times W + 1 \longrightarrow (iv)$$

2nd Part:-

- Given that,

  No load, i.e. $W=0$

  Now,

  Putting the value of $W=0$ in eqn no. (iv)

  $\Rightarrow P = \frac{3}{100} \times 0 - 1$

  $\Rightarrow P = 1N$ Ans

- Given that,

  load, $W = 100N$

  Now,

  Putting the value of $W=100N$ in eqn no. (i),

  $\Rightarrow P = \frac{3}{100} \times 100 + 1$

  $\Rightarrow P = 4N$ Ans

Q2 What load will be lifted by an effort of 12N, if the V.R. is 18 and efficiency of the machine at this load is 60%.

If the machine has a constant frictional resistance, determine the law of machine.

Soln; Given,

Effort applied; $P = 12N$

V.R. $= 18$

Efficiency, $\eta = 60\%$

$= 0.6$

Let,

$W =$ wt. lifted.

For simple lifting machine, WKT,

$M.A. = \frac{W}{P}$

$= \frac{W}{12N}$

Efficiency, $\eta = \frac{M.A.}{V.R}$ $\Rightarrow 0.6 = \frac{W/12}{18}$ $\Rightarrow W = 129.6N$ Ans

WKT,

Effort lost in friction, $F_{effort} = \cancel{\frac{P_x W}{V.R.}}\ P - \frac{W}{V.R.}$

$= 12 - \frac{129.6}{18}$

$= 4.8.$

Given that Frictiond resistance is constant,

$\therefore F_{effort} = C = 4.8$

WKT,

The law of machine is,

$P = mW + C \longrightarrow (1)$

Putting the values of P&W,C in eqⁿ ①,

$\Rightarrow 12 = m \times 129.6 + 4.8$

$\Rightarrow m = 0.05$

$\therefore$ The law of machine is:-

$P = 0.05\,W + 4.8$

Q3. A drum weight 6kg and holding 40kg of water is to be raised from a well by means of wheel and axle, machine. The axle is 10cm diameter and wheel is 40cm diameter. If a force of 12kg is to be applied to the wheel. Find:-

- M.A.
- V.R.
- Efficiency.

Given,

Wt. of drum, $W_1 = 6Kg$.

wt. of holding water, $W_2 = 40Kg$

$\therefore$ Total wt. to be lifted $W = W_1 + W_2$

$= 6 + 40$

$= 46Kg.$

Diameter of axle, $d = 10cm$

Diameter of wheel, $D = 40cm$.

Applied Force, $P = 12kg$.

For simple wheel and axle machine, WKT,

$* \; M.A. = \frac{W}{P}$ $\quad * \; V.R. = \frac{D}{d}$ $\quad * \; \text{Efficiency}, \eta = \frac{M.A}{V.R.} \times 100\%$

$= \frac{46 kg}{12 kg}$ $\quad = \frac{40 cm}{10 cm}$ $\quad = \frac{3.83}{4} \times 100\%$

$= 3.83.$ $\quad = 4.$ $\quad = 95.7\%.$

Q4. In a wt. lifting machine, whose V.R. is 20. A wt. of 1KN can be raised by an effort of 80N. If the effort is removed, show that the machine can work in the reverse direction.

Given,

$V.R = 20$

Weight lifted, $W = 1KN. = 1000N$

Effort Applied, $P = 80N$

Let

Efficiency $= \eta$

For a simple lifting Machine,

WKT,

$M.A. = \frac{W}{P}$

$= \frac{1000N}{80N}$

$= 12.5$

Now

$\text{Efficiency}, \eta = \frac{M.A.}{V.R.} \times 100\%$

$= \frac{12.5}{20}$

$= 62.5\%$

Here, efficiency value bought 62.5%. i.e., more than 50%.

Therefore the machine is a reversible machine and the machine can work in the reverse direction.

... lifting machine an effort of 50N is required to lift a load. The distance moved by the load and effort are 20mm and 500mm respectively. Determine the magnitude of the load if, the efficiency of the machine is 80%.

Given

Effort Applied, $P = 50N$

Distance moved by the load, $x = 20mm$

" " " effort, $y = 500mm$

Efficiency, $\eta = 80\% = 0.8$.

For a Simple Lifting Machine, WKT,

$$M.A. = \frac{W}{P} = \frac{W}{50N}$$

$$V.R = \frac{y}{x} = \frac{50\cancel{0}\ mm}{2\cancel{0}\ mm} = 25$$

$\therefore$ Efficiency, $\eta = \frac{M.A.}{V.R}$

$$0.8 = \frac{\frac{W}{50}}{25}$$

$$\frac{W}{50} = 0.8 \times 25$$

$$W = 0.8 \times 25 \times 50$$

$$W = 1000N$$

Q6. A load of 120N is raised by means of a certain wt. lifting machine through a distance of 200mm. If the effort applied is 20N and has move through a distance of 1.5m, Find the efficieny of the machine.

A) Given

Load lifted, $W = 120N$.

Distance moved by the load, $x = 200mm$.

Effort applied, $P = 20N$.

Distance moved by the effort, $y = 1.5m = 1500mm$

For a simple lifting machine, WKT,

$M.A. = \frac{W}{P}$ $\quad V.R. = \frac{y}{x}$ $\quad$ Efficiency, $\eta = \frac{M.A.}{V.R.} \times 100\%$

$= \frac{120N}{20N}$ $\quad = \frac{1500mm}{200mm}$ $\quad = \frac{6}{7.5} \times 100\%$

$= 6$ $\quad = 7.5$ $\quad = 80\%$.

Q7. A screw jack has a thread of 10mm pitch. What effort applied at the end of a handle 400mm long, will be required to lift a load of 2KN, if the efficiency at this load is 45%.

⇒ Given,

Pitch of thread, $p = 10mm$

length of Handle, $l = 400mm$

Weight lifted, $W = 2KN = 2000N$.

Efficiency, $\eta = 45\% = 0.45$

Let, P = Effort applied.

For a simple screw jack, we know that,

$V.R. = \frac{2\pi l}{p}$ $\quad M.A. = \frac{W}{P}$

$= \frac{2 \times 3.14 \times 400mm}{10mm}$ $\quad = \frac{2000N}{P}$

$V.R. = \frac{6.28 \times 400mm}{10mm}$ $\quad$ Efficiency, $\eta = \frac{M.A.}{V.R.}$

$= 251.20$ $\quad 0.45 = \frac{\frac{2000}{P}}{251.2}$

$\Rightarrow \frac{2000}{P} = 0.45 \times 251.2$

$\Rightarrow P = \frac{2000}{0.45 \times 251.2}$

$\Rightarrow P = 17.7N$.

Q8. In a lifting machine an effort of 500N is to be move by a distance of 20m to raise a load of 10000N by a distance of 0.8m., determine the M.A., V.R. and Efficiency (η).

Also determine the ideal effort and effort lost in friction.

⇒ Given,

Effort applied, $P = 500N$.

Distance moved by the effort, $y = 20m$.

Load lifted, $W = 10000N$.

Distance moved by the load, $x = 0.8m$.

For a S.L.M., WKT

$$M.A. = \frac{W}{P} = \frac{10000N}{500N} = 20$$

$$V.R. = \frac{y}{x} = \frac{20m}{0.8m} = 25$$

$$\text{Efficiency}, \eta = \frac{M.A.}{V.R.} \times 100\% = \frac{20}{25} \times 100\% = 80\%.$$

2nd Part,

Let,

P_1 = ideal effort.

WKT,

The efficiency of an ideal machine is 100%. = 1

WKT,

$$M.A. = \frac{W}{P_1} = \frac{10000N}{P_1}$$

V.R. = 25.

$$\text{Efficiency}, \eta = \frac{M.A.}{V.R.}$$

$$1 = \frac{\frac{10000}{P_1}}{25}$$

$P_1 = 400N$

Again,

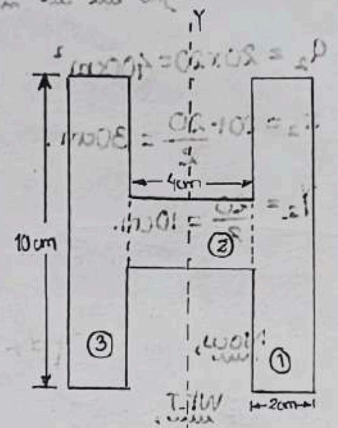
WKT,

$$\text{Effort lost in friction}, F_{effort} = P - \frac{W}{V.R.}$$

$$= 500 - \frac{10000}{25}$$

$$= 100N$$

Q. Find the C.G. of the H-section with dimensions, 10cm x 4cm x 2cm.



⇒ As the section is symmetrical along Y-Y axis

Therefore, the C.G. will lie on the axis.

Split up the section into three rectangle — ①, ② & ③.

Let, the bottom face be the axis of reference.

For rectangle ①, we have,

$\therefore a_1 = 10 \times 2 = 20cm^2$

$Y_1 = \frac{10}{2} = 5cm.$

From rectangle ②, we have,

$a_2 = 4 \times 2 = 8cm^2.$

$Y_2 = \frac{2}{2} = 1cm.$

From rectangle ③, we have,

$a_3 = 10 \times 2 = 20cm^2$

$Y_3 = \frac{10}{2} = 5cm.$

Now,

WKT,

The distance between the C.G. and the axis of reference is,

$$\Rightarrow \bar{Y} = \frac{a_1Y_1 + a_2Y_2 + a_3Y_3}{a_1 + a_2 + a_3}$$

$$\Rightarrow \bar{Y} = \frac{20 \times 5 + 8 \times 1 + 20 \times 5}{20 + 8 + 20}$$

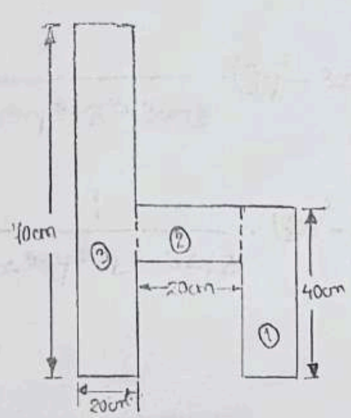
$$\Rightarrow \bar{Y} = 4.33cm$$

Q. Find the position of C.G. for plane lamina shown in Fig. 1 below.

⇒ The lamina is unsymmetrical along both the axis.

Split up the lamina into three rectangles.

Let, the bottom face and the left face, be the axis of reference.



From rectangle ① we have,

$a_1 = 40 \times 20 = 800cm^2$

$x_1 = 20 + 20 + \frac{20}{2} = 50cm$

$Y_1 = \frac{40}{2} = 20cm$

From rectangle (2), we have,

$a_2 = 20\times20 = 400cm^2$

$x_2 = 20+\frac{20}{2} = 30cm.$

$Y_2 = \frac{20}{2} = 10cm.$

From rectangle (3), we have,

$a_3 = 70\times20 = 1400\,cm^2.$

$x_3 = \frac{20}{2} = 10cm$

$Y_3 = \frac{70}{2} = 35cm.$

Now,

WKT,

The distance between the C.G and the axis of references are:-

$$\bar{X} = \frac{a_1x_1+a_2x_2+a_3x_3}{a_1+a_2+a_3}$$

$$= \frac{800\times50+400\times30+1400\times10}{800+400+1400}$$

$$= 25.38cm.$$

$$\bar{Y} = \frac{a_1Y_1+a_2Y_2+a_3Y_3}{a_1+a_2+a_3}$$

$$= \frac{800\times20+400\times10+1400\times35}{800+400+1400}$$

$$= 26.53cm.$$

Q. A solid hemisphere of diameter 16cm is placed on top of a solid cylinder whose diameter is also 16cm. The height of the cylinder is 20cm.

Partial Differentiation

Date:- 06.08.2025

constant

$[y\text{-const.}]\ \frac{\partial}{\partial x}(x^2+y^2) = 2x+0$

$[x\text{-const}]\ \frac{\partial}{\partial y}(x^2+y^2) = 0+2y$

Q. Find the 1st order Partial Derivative of $ax^2+2hxy+by^2$.

Let, $u = ax^2+2hxy+by^2$

$$\frac{\partial u}{\partial x} = a\cdot 2x + 2hy + 0$$

$$\frac{\partial u}{\partial y} = 0 + 2hx + 2by.$$

Q. $f = x^y + y^x$. Find $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$.

$\Rightarrow \frac{\partial f}{\partial x} = y\cdot x^{y-1} + f\log y.$

$\Rightarrow \frac{\partial f}{\partial y} = f\log x + x\cdot y^{x-1}$

$y = 5^x$

$\log y = \log 5^x$

$\frac{1}{y}\frac{dy}{dx} = x\log 5$

$\frac{dy}{dx} = y\log 5$

Q. $u = \log(x^2+y^2)$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}\cdot 2x$

$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{x^2+y^2}\cdot 2y.$

Q. If $u = \log(x^3+y^3+z^3-3xyz)$.

P.T. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{x^3+y^3+z^3-3xyz}\cdot(3x^2-3yz)$

$\Rightarrow \frac{\partial u}{\partial y} = \frac{1}{x^3+y^3+z^3-3xyz}\cdot(3y^2-3xz)$

$\Rightarrow \frac{\partial u}{\partial z} = \frac{1}{x^3+y^3+z^3-3xyz}\cdot(3z^2-3xy)$

Now,

Note:-
$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z} = \frac{3}{x+y+z}$$

LHS.

$$\Rightarrow \frac{(3x^2-3yz)}{x^3+y^3+z^3-3xyz} + \frac{3y^2-3xz}{x^3+y^3+z^3-3xyz} + \frac{3z^2-3xy}{x^3+y^3+z^3-3xyz}$$

$$\Rightarrow \cancel{\frac{3[(x^2-yz)+(y^2-xz)+(z^2-xy)]}{}}$$

$$\Rightarrow \frac{1}{x^3+y^3+z^3-3xyz}(3x^2-3yz) + \frac{1}{x^3+y^3+z^3-3xyz}(3y^2-3xz) + \frac{1}{x^3+y^3+z^3-3xyz}(3z^2-3xy).$$

$$\Rightarrow \frac{1}{x^3+y^3+z^3-3xyz}(3x^2-3yz+3y^2-3xz+3z^2-3xy).$$

$$\Rightarrow \frac{3\cancel{(x^2+y^2+z^2-yz-xz-xy)}}{(x+y+z)\cancel{(x^2+y^2+z^2-xy-yz-zx)}}$$

$$\Rightarrow \frac{3}{(x+y+z)}$$