TRIGONOMETRY

$n\theta = \dfrac{1}{\operatorname{Cosec}\theta} \Rightarrow \operatorname{Sin}\theta \cdot \operatorname{Cosec}\theta = 1$

$s\theta = \dfrac{1}{\operatorname{Sec}\theta} \Rightarrow \operatorname{Cos}\theta \cdot \operatorname{Sec}\theta = 1$

$n\theta = \dfrac{1}{\operatorname{Cot}\theta} \Rightarrow \tan\theta \cdot \operatorname{Cot}\theta = 1$

ndard result :–

① $\sin^2\theta + \cos^2\theta = 1$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\sin\theta = \sqrt{1-\cos^2\theta}$

② $1+\tan^2\theta = \sec^2\theta$

$\sec\theta = \sqrt{1+\tan^2\theta}$

$\tan\theta = \sqrt{\sec^2\theta - 1}$

$\sec^2\theta - \tan^2\theta = 1$

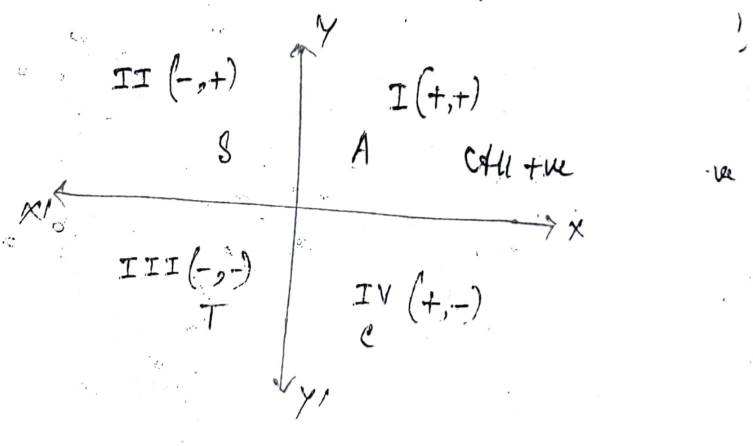
③ $1+\cot^2\theta = \operatorname{cosec}^2\theta$

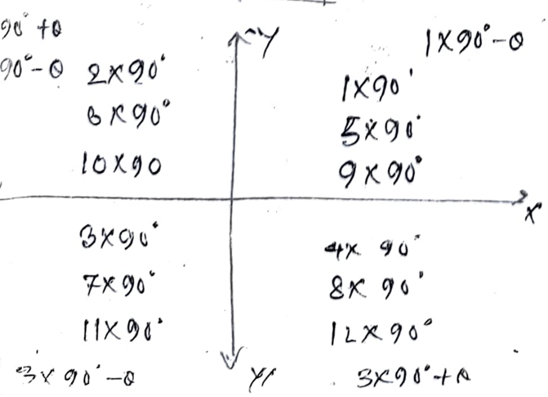
$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$

$\cot\theta = \sqrt{\operatorname{cosec}^2\theta - 1}$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

0	30°	45	60	90°
0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1





$\theta < \pi/2$

$90-\theta$ – 1st Q.

$90°+\theta$ – 2nd Q.

$1\times 90^\circ+\theta$	$1\times 90^\circ-\theta$
$2\times 90^\circ-\theta$	$4\times 90^\circ+\theta$
$3\times 90^\circ-\theta$	$3\times 90^\circ+\theta$
$2\times 90^\circ+\theta$	$4\times 90^\circ-\theta$

$1\times 90^\circ-\theta = 1$st Q

$1\times 90^\circ+\theta = 2$nd $= (1+1)$ Q.

$4\times 90^\circ-\theta = 4$th Q.

$4\times 90^\circ+\theta = 5$th $= 1$st Q

$19\times 90^\circ+\theta = 20 = \frac{20}{4} = 4$th

$23\times 90^\circ-\theta = \frac{23}{4} = 3$rd.

$25\times 90^\circ-\theta = \frac{25}{4} = 1$st

$33\times 90^\circ+\theta = \frac{34}{4} = 2$nd.

$420^\circ = 5\times 90^\circ - 30^\circ = \frac{5}{4} = 1$st quadrant

$540^\circ = 90^\circ\times 6 = \frac{6}{4} = 2$nd.

Remainder, when we devide by 4 if 0 = ~~4th~~ 4th quadrant
1 = 1st "
2 = 2nd "
3 = 3rd "

* If coefficient of 90° is an odd then the trigonometric ratios are changed as:

Sin ⟷ Cos.
tan ⟷ Cot
Sec ⟷ Cosec.

But if the coefficient of 90° is an even then there is no change in trigonometric ratios.

i.e. Sin ⟷ Sin, tan ⟷ tan, Cos ⟷ Cos.

$\sin 120^\circ = \sin(90^\circ+30^\circ) = \cos 30^\circ = \sqrt{3}/2$

(Quadrant x Put the sign x Ratio Q. (odd/even) $\frac{S \mid A}{T \mid C}$)

$\cot 150^\circ = \cot(90^\circ+60^\circ) = -\tan 60^\circ = -\sqrt{3}$

$\sec 300^\circ = \sec(3\times 90^\circ+30^\circ) = \operatorname{cosec} 30^\circ = 2$

$\tan 420^\circ = \tan(4\times 90^\circ+60^\circ) = \tan 60^\circ = \sqrt{3}$

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all the t-ratios of $90^\circ\pm\theta$, $180^\circ\pm\theta$; $270\pm\theta$; $360\pm\theta$.

$1\times 90^\circ-\theta$ = 1st Q

$\left.\begin{matrix}\sin\\ \cos\\ \tan\\ \cot\\ \sec\\ \operatorname{cosec}\end{matrix}\right\}(1\times 90^\circ-\theta)\begin{matrix}=\cos\theta\\ =\sin\theta\\ =\cot\theta\\ =\tan\theta\\ =\operatorname{cosec}\theta\\ =\sec\theta.\end{matrix}$

$1\times 90^\circ+\theta$ = 2nd

$\left.\begin{matrix}\sin\\ \cos\\ \tan\\ \cot\\ \sec\\ \operatorname{cosec}\end{matrix}\right\}(1\times 90^\circ+\theta)\begin{matrix}=\cos\theta\\ =-\sin\theta\\ =-\cot\theta\\ =-\tan\theta\\ =-\operatorname{cosec}\theta\\ =\sec\theta\end{matrix}$

$= \frac{1}{2\lfloor n-2} + \frac{1}{\lfloor n-1}$

$T_1 = \frac{1}{2\lfloor 1-2} + \frac{1}{\lfloor 1-1} = 0 + \frac{1}{1}$

$T_2 = \frac{1}{2\lfloor 0} + \frac{1}{\lfloor 2-1} = \frac{1}{2} + \frac{1}{\lfloor 1}$

$T_3 = \frac{1}{2\lfloor 1} + \frac{1}{\lfloor 2}$, $T_4 = \frac{1}{2\lfloor 2} + \frac{1}{\lfloor 3}$

$+T_2+T_3+\ldots$

$= \left(0+\frac{1}{2}+\frac{1}{2\lfloor 1}+\ldots\right)+\left(1/1 + 1/\lfloor 1 + 1/\lfloor 2 + 1/\lfloor 3 + \ldots\right)$

$= \frac{1}{2}\left(1+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\ldots\right)+e$

$= 1/2\, e + e = 3/2\, e$

Binomial theorem

Bi – Two nomial = Number.

An expression ~~containing~~ containing two terms ~~containing~~ adding or substracting is called a binomial.

e.g. $a+b$, $x-1/a$, $3x-4/x^2$, $a+n$ etc.

(1) ${}^nC_r$ or ${}_nC_r$ or $C_{(n,r)}$

$${}^nC_r = \frac{n(n-1)(n-2)(n-3)\ldots(n-r+1)}{1.2.3.4.\ldots r}\quad (n\geq r)$$

$$\text{or } {}^nC_r = \frac{\lfloor n}{\lfloor r\,\lfloor n-r}$$

Q. Find the value of

$${}^nC_2 = \frac{n(n-1)}{\lfloor 2}$$

$${}^nC_2 = \frac{\lfloor n}{\lfloor 2\,\lfloor n-2} = \frac{n(n-1)\lfloor n-2}{\lfloor 2\,\lfloor n-2}$$

$${}^{5}C_{5} = \frac{5.4.3.2.1}{\lfloor 5 \; \lfloor 5-5} = \frac{\lfloor 5}{\lfloor 5 \; \lfloor 0} = 1$$

$${}^{n}C_{n-1} = \frac{\lfloor n}{\lfloor n-1 \; \lfloor n-(n-1)}$$

$$= \frac{\lfloor n}{\lfloor n-1 \; \lfloor 1}$$

$$= \frac{n \lfloor n-1}{\lfloor n-1} = n$$

$${}^{n}C_{1} = n$$

$$\therefore \; {}^{n}C_{r} = {}^{n}C_{n-r}$$

$${}^{28}C_{24} = {}^{28}C_{4} = \frac{28.27.26.25}{\lfloor 4}$$

$${}^{n}C_{0} = 1 = {}^{n}C_{n}$$

$2\times 90^\circ - \theta$ = 2nd

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (2\times 90^\circ - \theta) = \begin{array}{l} +\sin\theta \\ -\cos\theta \\ -\tan\theta \\ -\cot\theta \\ -\sec\theta \\ +\operatorname{cosec}\theta \end{array}$$

3rd

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (2\times 90^\circ + \theta) = \begin{array}{l} -\sin\theta \\ -\cos \\ +\tan \\ +\cot \\ -\sec \\ -\operatorname{cosec} \end{array}$$

$3\times 90^\circ - \theta$ = 3rd

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (3\times 90^\circ - \theta) = \begin{array}{l} -\cos \\ -\sin \\ +\cot \\ +\tan \\ -\operatorname{cosec} \\ -\sec \end{array}$$

4th

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (3\times 90^\circ + \theta) = \begin{array}{l} -\cos \\ +\sin \\ -\cot \\ -\tan \\ +\operatorname{cosec} \\ -\sec \end{array}$$

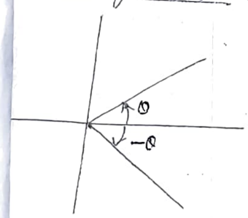
4th

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (4\times 90^\circ - \theta) = \begin{array}{l} -\sin \\ +\cos \\ -\tan \\ -\cot \\ +\sec \\ -\operatorname{cosec} \end{array}$$

1st

$$\left.\begin{array}{l} \sin \\ \cos \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}\right\} (4\times 90^\circ + \theta) = \begin{array}{l} \sin \\ \cos\theta \\ \tan \\ \cot \\ \sec \\ \operatorname{cosec} \end{array}$$

of angle $(-\theta)$:-



Since $-\theta$ lies in the 4th Q and in the 4th quadrant Cos and Sec alone +ve

$$\therefore \begin{array}{lcl} \sin(-\theta) & = & -\sin\theta \\ \cos(-\theta) & = & \cos\theta \\ \tan(-\theta) & = & -\tan\theta \\ \cot(-\theta) & = & -\cot\theta \\ \sec(-\theta) & = & \sec\theta \\ \operatorname{cosec}(-\theta) & = & -\operatorname{cosec}\theta \end{array}$$

Find the value of $\sin(-660^\circ)$

$$\sin(-660^\circ) = -\sin(660^\circ)$$

$$= -\sin(7\times 90^\circ + 30^\circ)$$

$$= -(-\cos 30^\circ)$$

$$= \frac{\sqrt{3}}{2}$$

$$\frac{720}{660} \quad 60$$

(ii) $\cos\left\{-11\times\frac{\pi}{4}\right\}$

$= \cos\frac{11\pi}{4}$

$= \cos 495$

$= \cos(5\times 90^\circ + 45)$

$= -\cos 45^\circ = -\frac{1}{\sqrt{2}}$

(iii) $\tan(-1125^\circ)$

$= -\tan(90^\circ\times 12 + 45^\circ)$

$= -\tan 45^\circ = -1$

(iv) $\sin 780^\circ$ and $\cos(-780^\circ)$.

$= \sin(8\times 90^\circ + 60^\circ)$

$= \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos(-780^\circ)$

$= \cos(780^\circ)$

$= \cos(8\times 90^\circ + 60^\circ)$

$= \cos 60^\circ = \frac{1}{2}$

Q. Eliminate θ :—

(i) $x = a\sec\theta$, $y = b\tan\theta$.

$\Rightarrow \frac{x}{a} = \sec\theta$, $\frac{y}{b} = \tan\theta$.

$\sec^2\theta - \tan^2\theta = \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2$

$\Rightarrow 1 = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

Q. Find x if

$\tan^2 45^\circ - \cos^2 60^\circ = x\sin 45^\circ \cos 45^\circ \tan 60^\circ$

$\Rightarrow (1)^2 - \left(\frac{1}{2}\right)^2 = x\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\times\sqrt{3}$

$\Rightarrow \frac{4-1}{4} = x\cdot\frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

$\begin{array}{r} 540 \\ 495 \\ \hline 5 \end{array}$

Q. Prove that $\sin^2 36^\circ + \sin^2 54^\circ = 1$

$\sin^2 36^\circ + \{\sin(90^\circ - 36^\circ)\}^2$

$= \sin^2 36^\circ + \cos^2 36^\circ$

$= 1$ Proved

Q. $\sin^2 49^\circ + \sin^2 41^\circ = 1$

L.H.S. $= \sin^2 49^\circ + \sin^2 41^\circ$

$= \sin^2 49^\circ + \cos^2 49^\circ$

$= 1 =$ R.H.S

Q. Find the value of (i) $\cos(-225^\circ)$

$= \cos(90^\circ\times 2 + 45^\circ)$

$= -\cos 45^\circ = -\frac{1}{\sqrt{2}}$

Q. P.T. $\sin 420^\circ \times \cos 390^\circ + \cos(-300^\circ)\sin(-330^\circ) = 1$

L.H.S $= \sin 420^\circ.\cos 390^\circ + \cos(-300^\circ).\sin(-330^\circ)$

$= \sin(90^\circ\times 5 - 30^\circ).\cos(4\times 90^\circ + 30^\circ) + \cos(3\times 90^\circ + 30^\circ)$ $\{-\sin(4\times 90^\circ - 30^\circ)\}$

$= \cos 30^\circ . \cos 30^\circ + (\sin 30^\circ).(\sin 30^\circ)$

$= \cos^2 30^\circ + \sin^2 30^\circ$

$= 1 =$ R.H.S

(1) If $\cot\theta = 15/8$ find $\cos\theta$ and $\csc\theta$.

(2) Find $\sin(-1230^\circ)$ and $\tan(-2010^\circ)$

(3) $\operatorname{cosec}(-7\times\pi/4)$

(1) $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$= 1 + \left(\frac{15}{8}\right)^2$

$\tan\theta = \frac{8}{15}$

$1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow 1 + \frac{64}{225} = \sec^2\theta$

$\Rightarrow \frac{289}{225} = \sec^2\theta$

$\Rightarrow \sec\theta = \frac{\sqrt{289}}{15}$

$\therefore \cos\theta = \frac{15}{\sqrt{289}}$

$$\begin{array}{r} 1230 \\ 1080 \\ \hline 150 \end{array}$$

② $\sin(-1230^\circ)$

$= -\sin(90^\circ \times 12 + 150^\circ)$

$= -\sin 150^\circ$

$= -\sin(90^\circ + 60^\circ)$

$= -\cos 60^\circ = -\frac{1}{2}$.

$$\begin{array}{r} 2010 \\ 1800 \\ \hline 210 \end{array}$$

$\tan(2010^\circ) = \tan(90^\circ \times 20 + 210^\circ)$

$= \tan 210^\circ$

$= \tan(90^\circ \times 3 - 60^\circ)$

$= \cot 60^\circ = \frac{1}{\sqrt{3}}$

③ $\operatorname{cosec}(-7\pi/4)$

$= \operatorname{cosec}(-7 \times 45^\circ)$

$= -\operatorname{cosec}(315^\circ)$

$= -\operatorname{cosec}(4 \times 90^\circ - 45^\circ)$

$= +\operatorname{cosec} 45^\circ$

$= \sqrt{2}$

$\operatorname{cosec}(2040^\circ)$

$$\begin{array}{r} 2040 \\ 1800 \\ \hline 240 \end{array}$$

$\operatorname{cosec}(90^\circ \times 20 + 240^\circ)$

$\operatorname{cosec} 240^\circ$

~~$= \sec(90 \times 3 - 30^\circ)$~~

~~$\operatorname{cosec} 30$~~ $= \operatorname{cosec}(90^\circ \times 3 - 30^\circ)$

$$\begin{array}{r} 1305 \\ 1080 \\ \hline 225 \end{array}$$

$= \cancel{\sin 30^\circ}\ -\sec 30^\circ$

$= -2/\sqrt{3}$

④ $\sec(-1305^\circ)$

$= \sec(90^\circ \times 12 + 225^\circ)$

$= \sec 225^\circ$

$= \sec(90^\circ \times 3 - 45^\circ)$

$= -\operatorname{cosec} 45^\circ = -\sqrt{2}$

Compound Angle :-

Algebric sum or difference of two or more angles called a compound angle.

$A+B$, $A-B$, $A+B+C$, $A-B+C$ etc.

A and B are two +ve acute angles then we have the following add^n and subtraction formulae.

Addition formulae

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$.

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\cot(A+B) = \frac{1 - \tan A \tan B}{\tan A + \tan B}$

$= \frac{\cot A \cdot \cot B - 1}{\cot A + \cot B}$.

Substraction formula:-

⑤ $\sin(A-B) = \sin A \cos B - \cos A . \sin B$.

⑥ $\cos(A-B) = \cos A . \cos B + \sin A \sin B$

⑦ $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A . \tan B}$

⑧ $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$.

Q. Find the value of

(i) $\cos 75° = \cos(45° + 30°)$

$= \cos 45° . \cos 30° - \sin 45° . \sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(ii) $\sec 75° = \dfrac{1}{\cos(45°+30°)}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}-1}$

(iii) $\operatorname{cosec} 75° = \dfrac{1}{\sin(45°+30°)}$

$= \dfrac{1}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}}$

$= \dfrac{2\sqrt{2}}{\sqrt{3}+1}$

(vi) $\cot 75° = \dfrac{\cot 45 . \cot 30° - 1}{\cot 45° + \cot 30°}$

$= \dfrac{1 . \sqrt{3} - 1}{1 + \sqrt{3}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

(iv) $\sin 15° = \sin 45° . \cos 30° - \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

(v) $\cos 15° = \cos 45° . \cos 30° + \sin 45° . \sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

(vi) $\tan 15° = \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° . \tan 30°}$

$= \dfrac{1 - 1/\sqrt{3}}{1 + 1 . 1/\sqrt{3}}$

$= \dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

P.T. $\cos 75° = \sin 15°$

From (i) and (iv)

$\sin(90° - 75°)$

$\Rightarrow \cos 75 = \sin 15°$

P.T. $\dfrac{\sin(A-B)}{\cos A . \cos B} + \dfrac{\sin(B-C)}{\cos B . \cos C} + \dfrac{\sin(C-A)}{\cos C . \cos A} = 0$

L.H.S $= \dfrac{\sin(A-B)}{\cos A . \cos B} + \dfrac{\sin(B-C)}{\cos B \cos C} + \dfrac{\sin(C-A)}{\cos C . \cos A}$

$= \dfrac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \dfrac{\sin B . \cos C - \cos B \sin C}{\cos B . \cos C} + \dfrac{\sin C \cos A - \cos A \sin C}{\cos C . \cos A}$

$\tan A - \tan B + \tan B - \tan C + \tan C - \tan A$

$0 = R.H.S$

If $\sin A = 3/5$ $\cos B = 12/13$

P.T. (i) $\sin(A-B) = 16/65$

(ii) $\cos(A+B) = \dfrac{33}{65}$

$\sin(A-B) = \frac{3}{5} . \frac{12}{13} - \cos A . \sin B$

$= \frac{36}{65} - \sqrt{1-(3/5)^2}\sqrt{1-(12/13)^2}$

$$= \frac{36}{65} - \sqrt{\frac{25-9}{25}}\sqrt{\frac{169-144}{169}}$$

$$= \frac{36}{65} - \frac{4}{\cancel{5}} \times \frac{\cancel{5}}{13}$$

$$= \frac{36-20}{65} = \frac{16}{65}$$

(ii)

$$\cos(A+B) = \cos A . \cos B - \sin A \sin B$$

$$= \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13}$$

$$= \frac{48-15}{65}$$

$$= \frac{33}{65}$$

$$\left(\cos A = \frac{4}{5},\ \sin B = \frac{5}{13}\right)$$

(5) P.T.

$$\frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ} = \tan 37^\circ$$

$$L.H.S = \frac{\cos 8^\circ - \sin 8}{\cos 8 + \sin 8^\circ}$$

$$= \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ . \tan 8^\circ}$$

$$= \tan(45^\circ - 8^\circ)$$

$$= \tan 37^\circ$$

$$= R.H.S$$

(6). P.T.

(i) $\sin(A+B)\sin(A-B) = \sin^2 A - \cancel{\cos^2} \sin^2 B = \cos^2 B - \cos^2 A$

(ii) $\cos(A+B).\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

$$\tan(A+B)\tan(A-B) = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$$

$$\sin(A+B).\sin(A-B) = (\sin A \cos B + \cos A \sin B)(\sin A . \cos B - \cos A \sin B)$$

$$= \sin^2 A . \cos^2 B - \cos^2 A . \sin^2 B$$

$$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) . \sin^2 B$$

$$= \sin^2 A - \cancel{\sin^2 A \sin^2 B} - \sin^2 B + \cancel{\sin^2 A \sin^2 B}$$

$$= \cos^2 B - \cos^2 A = \sin^2 A - \sin^2 B$$

$$\cos(A+B).\cos(A-B) = (\cos A . \cos B - \sin A \sin B)(\cos B . \cos A + \sin A . \sin B)$$

$$= \cos^2 B . \cos^2 A - \sin^2 A . \sin^2 B$$

$$= (1 - \sin^2 B).\cos^2 A - (1 - \cos^2 A)\sin^2 B$$

$$= \cos^2 A - \cancel{\sin^2 B . \cos^2 A} - \sin^2 B + \cancel{\sin^2 B . \cos^2 A}$$

$$= \cos^2 A - \sin^2 B$$

$$= \cos^2 B - \sin^2 A$$

$$\tan(A+B).\tan(A-B) = \frac{\sin(A+B)}{\cos(A+B)} \times \frac{\sin(A-B)}{\cos(A-B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$$

If $\tan(A+B) = p$, $\tan(A-B) = q$

nd $\tan 2A$.

$$\tan\{(A+B) + (A-B)\} = \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) . \tan(A-B)}$$

$$\Rightarrow \tan 2A = \frac{p+q}{1-pq}$$

Q. Find (i) $\sin(A+B+C)$

(ii) $\cos(A+B+C)$

(iii) $\tan(A+B+C)$

(iv) $\cot(A+B+C)$

$\sin(A+B+C) = \sin(A+B)\cos C + \cos(A+B).\sin C$

$= (\sin A.\cos B + \cos A.\sin B)\cos C + (\cos A\cos B - \sin A\sin B).\sin C$

$= \sin A.\cos B\cos C + \cos A\sin B\cos C + \cos A.\cos B\sin C$

$- \sin A.\sin B\sin C$

(ii) $\cos(A+B+C) = \cos(A+B).\cos C - \sin(A+B).\sin C$

$= (\cos A\cos B - \sin A.\sin B)\cos C - (\sin A\cos B + \cos A\sin B)\sin C$

$= \cos A.\cos B.\cos C - \sin A.\sin B.\cos C - \sin A.\cos B\sin C$

$- \cos A.\sin B.\sin C$

(iii) $\tan(A+B+C) = \dfrac{\tan(A+B) + \tan C}{1 - \tan(A+B).\tan C}$

$= \dfrac{\dfrac{\tan A + \tan B}{1 - \tan A.\tan B} + \tan C}{1 - \dfrac{\tan A + \tan B}{1 - \tan A\tan B}\tan C}$

$= \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B.\tan C - \tan C\tan A}$

(iv) $\cot(A+B+C) = \dfrac{1 - \tan A\tan B - \tan B\tan C - \tan C.\tan A}{\tan A + \tan B + \tan C - \tan A\tan B\tan C}$

$= \dfrac{1 - \dfrac{1}{\cot A\cot B} - \dfrac{1}{\cot B\cot C} - \dfrac{1}{\cot C\cot A}}{\dfrac{1}{\cot A} + \dfrac{1}{\cot B} + \dfrac{1}{\cot C} - \dfrac{1}{\cot A\cot B\cot C}}$

$= \dfrac{\cot A.\cot B.\cot C - \cot A - \cot B - \cot C}{\cot A.\cot B + \cot B\cot C + \cot C\cot A - 1}$

If $A+B+C = \pi$. P.T. $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

$\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\tan B\tan C}{1 - \tan A\tan B - \tan B\tan C - \tan C\tan A}$

$\Rightarrow \tan(\pi) = \ldots$

$\Rightarrow 0 = \ldots$

$\therefore \tan A + \tan B + \tan C - \tan A.\tan B.\tan C = 0$

$\Rightarrow \tan A + \tan B + \tan C = \tan A\tan B.\tan C$

P.T. $\cos(60°-A)\cos(30°-B) - \sin(60°-A)\sin(30°-B) = \sin(A+B)$

L.H.S $= \cancel{(\cos 60°\cos A + \sin 60°.\sin A)}$

$= \cos(60°-A)\cos(30°-B) - \sin(60°-A)\sin(30°-B)$

$= \cos\{60 - A + 30° - B\}$

$= \cos\{90° - (A+B)\}$

$= \sin(A+B)$

$=$ R.H.S

P.T. $\dfrac{\sin(A-B)}{\cos A.\cos B} + \dfrac{\sin(B-C)}{\cos B\cos C} + \dfrac{\sin(C-A)}{\cos C.\cos A} = 0$

$= \cancel{\tan A} - \cancel{\tan B} + \cancel{\tan B} - \cancel{\tan C} + \cancel{\tan C} - \cancel{\tan A}$

$= 0$

Q. If $A+B=\pi/4$, P.T.

$(1+\tan A)(1+\tan B)=2$.

$L.H.S = \left\{1+\tan\left(\pi/4 - B\right)\right\}(1+\tan B)$

$= \left(1+\dfrac{1-\tan B}{1+\tan B}\right)\times(1+\tan B)$

$= \dfrac{1+\cancel{\tan B}+1-\cancel{\tan B}}{\cancel{1+\tan B}}\cancel{(1+\tan B)}$

$= 2 = R.H.S$

Q. P.T.

$\tan 20^\circ + \tan 25^\circ + \tan 25^\circ \tan 20^\circ = 1$

We know

$45^\circ = 20^\circ + 25^\circ$

$\Rightarrow \tan 45^\circ = \tan(20+25^\circ)$

$\Rightarrow 1 = \dfrac{\tan 20^\circ + \tan 25^\circ}{1-\tan 20^\circ \cdot \tan 25^\circ}$

$\Rightarrow 1-\tan 20^\circ \cdot \tan 25^\circ = \tan 20^\circ + \tan 25^\circ$

$\Rightarrow \tan 20^\circ + \tan 25^\circ + \tan 20^\circ \cdot \tan 25^\circ = 1$ Proved.

Q. $\tan(A+B)\tan(A-B) = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 B - \sin^2 A}$.

Q. $\tan^2 A - \tan^2 B = \dfrac{\sin(A+B)\sin(A-B)}{\cos^2 A \cdot \cos^2 B}$

$R.H.S. = \dfrac{\sin(A+B)\sin(A-B)}{\cos^2 A \cdot \cos^2 B}$

$= \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A \cdot \cos^2 B}$

$= \tan^2 A \cdot \dfrac{1}{\cos^2 B} - \tan^2 B \cdot 1/\cos^2 A$

$= \tan^2 A \cdot \sec^2 B - \tan^2 B \cdot \sec^2 A$

$= \tan^2 A(1+\tan^2 B) - \tan^2 B(1+\tan^2 A)$

$= \tan^2 A + \tan^2 A \cdot \tan^2 B - \tan^2 B - \cancel{\tan^2 A \tan^2 B}$

$= \tan^2 A - \tan^2 B$

$= L.H.S$

P.T. $\dfrac{\tan(\alpha+\beta)-\tan\alpha}{1+\tan(\alpha+\beta)\tan\alpha} = \tan\beta$

$L.H.S = \tan(\alpha+\beta-\alpha)$

$= \tan\beta$

P.T.

$1+\tan\theta \cdot \tan 2\theta = \sec 2\theta$

$\Rightarrow L.H.S. =$

$= 1+\dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\sin 2\theta}{\cos 2\theta}$

$= \dfrac{\cos\theta \cdot \cos 2\theta + \sin\theta \cdot \sin\theta}{\cos\theta \cdot \cos 2\theta}$

$= \dfrac{\cos(2\theta-\theta)}{\cos\theta \cos 2\theta}$

$= \dfrac{\cos\theta}{\cos\theta \cdot \cos 2\theta}$

$= \sec 2\theta$

$= R.H.S$

If $A+B+C=\pi$ and $\cos A = \cos B \cdot \cos C$

then show that

$\tan A = \tan B + \tan C$

$\cos A = \cos B \cdot \cos C$

$\cos\{\pi-(B+C)\} = \cos B \cdot \cos C$

$\Rightarrow -\cos(B+C) = \cos B.\cos C$

$\Rightarrow -\cos B\cos C + \sin B\sin C = \cos B\cos C$

$\Rightarrow 2\cos B.\cos C = \sin B.\sin C$

$\Rightarrow 2 = \tan B.\tan C$

$A+B+C=\pi \Rightarrow A = \pi-(B+C) \longrightarrow (1)$

$\cos A = \cos B.\cos C \longrightarrow (2)$

$L.H.S = \tan A = \dfrac{\sin A}{\cos A}$

$= \dfrac{\sin(B+C)}{\cos B\cos C}$

$= \dfrac{\sin B.\cos C + \cos B\sin C}{\cos B\cos C}$

$= \tan B + \tan C = R.H.S$

Q. P.T.

$\cot\theta - \cot 2\theta = \operatorname{cosec} 2\theta$

$L.H.S = \cot\theta - \cot 2\theta$

$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\sin 2\theta}$

$= \dfrac{\cos\theta.\sin 2\theta - \sin\theta.\cos 2\theta}{\sin\theta\sin 2\theta}$

$= \dfrac{\sin(2\theta-\theta)}{\sin\theta\sin 2\theta}$

$= \operatorname{cosec} 2\theta$

P.T.

$\tan 20^\circ + \tan 25^\circ + \tan 20^\circ\tan 25^\circ = 1$

$\tan(A+B) + \tan(A-B) = \dfrac{\sin 2A}{\cos^2 A - \sin^2 B}$

$L.H.S. = \tan(A+B) + \tan(A-B)$

$= \dfrac{\tan A + \tan B}{1-\tan A\tan B} + \dfrac{\tan A - \tan B}{1+\tan A\tan B}$

$= \dfrac{\sin A.\cos B + \sin B\cos A}{\cos A\cos B - \sin A\sin B} + \dfrac{\sin A\cos B - \cos A\sin B}{\cos A\cos B + \sin A\sin B}$

$= \dfrac{\sin(A+B)}{\cos(A+B)} + \dfrac{\sin(A-B)}{\cos(A-B)}$

$= \dfrac{\sin\{(A+B+A-B)}{\cos(A+B)\cos(A-B)}$

$= \dfrac{\sin 2A}{\cos^2 A - \sin^2 B}$

Transformation of product into sums or difference

We know

(i) $\sin(A+B) = \sin A.\cos B + \cos A\sin B \longrightarrow (1)$

(ii) $\sin(A-B) = \sin A\cos B - \cos A\sin B \longrightarrow (2)$

(iii) $\cos(A+B) = \cos A.\cos B - \sin A\sin B \longrightarrow (3)$

(iv) $\cos(A-B) = \cos A\cos B + \sin A\sin B \longrightarrow (4)$

$\sin(A+B) + \sin(A-B) = 2\sin A.\cos B \longrightarrow (5)$

$\Rightarrow \sin(A+B) - \sin(A-B) = 2\cos A\sin B \longrightarrow (6)$

(iv) $\Rightarrow \cos(A+B) + \cos(A-B) = 2\cos A.\cos B \longrightarrow (7)$

(4) $\Rightarrow \cos(A+B) - \cos(A-B) = -2\sin A\sin B \longrightarrow (8)$

$\Rightarrow \cos(A-B) - \cos(A+B) = 2\sin A \cdot \sin B,$

Application :-

Express as sum or difference

(1) $\sin 30 \cdot \sin 50$

$= \frac{1}{2}(2\sin 30 \cdot \sin 50)$

$= \frac{1}{2}\{\cos(50-30) - \cos(50+30)\}$

$= \frac{1}{2}(\cos 20 - \cos 80)$

(2) $\cos 40^\circ \cdot \cos 80^\circ$

$= \frac{1}{2}\{2\cos 40^\circ \cdot \cos 80^\circ\}$

$= \frac{1}{2}\{\cos(80^\circ+40^\circ) + \cos(80^\circ-40^\circ)\}$

$= \frac{1}{2}(\cos 120^\circ + \cos 40^\circ)$

$= \frac{1}{2}\{-\frac{1}{2} + \cos 40^\circ\}$

$= -\frac{1}{4} + \cos 40^\circ/2$

(3) $\sin 20^\circ \cdot \sin 40 \sin 80^\circ$

$= \frac{1}{2}(2\sin 20^\circ \sin 40^\circ) \cdot \sin 80^\circ$

$= \frac{1}{2}\{\cos 20^\circ - \cos 60^\circ\} \sin 80^\circ$

$= \frac{1}{2}\cos 20^\circ \cdot \sin 80^\circ - \frac{1}{2}\times\frac{1}{2}\times\sin 80^\circ$

$= \frac{1}{4}(2\cos 20^\circ \sin 80^\circ) - \frac{1}{4}\times\sin 80^\circ$

$= \frac{1}{4}(\sin 100^\circ + \sin 60^\circ) - \frac{1}{4}\times\sin 80^\circ$

$= \frac{1}{4}\sin 100^\circ + \frac{\sqrt{3}}{8} - \frac{1}{4}\sin 80^\circ$

$= \cancel{\frac{1}{4}\,\sin(80^\circ+20^\circ) - \frac{\sqrt{3}}{8}\times\frac{\sqrt{3}}{2} - \frac{1}{4}\sin 80}$

$= \frac{1}{4}\sin(90^\circ+10^\circ) + \frac{\sqrt{3}}{8}\ \frac{3}{16} - \frac{1}{4}\sin 80^\circ$

$= \frac{1}{4}\cos 10^\circ - \frac{\sqrt{3}}{8} - \frac{1}{4}\cos 10^\circ = +\frac{\sqrt{3}}{8}$

(4) $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$

$L.H.S. = \frac{1}{2}(2\cos 20^\circ \cdot \cos 40^\circ) \cdot \frac{1}{2}(2\cos 60^\circ \cdot \cos 80^\circ)$

$= \frac{1}{2}\{\cos 60^\circ - \cos 20^\circ\} \cdot \frac{1}{2}(\cos 140^\circ - \cos 20^\circ)$

$= (\frac{1}{2}\times\frac{1}{2} - \frac{1}{2}\cos 20^\circ)(\frac{1}{2}\cos(180-40^\circ) - \cos 20^\circ)$

$= (\frac{1}{4} - \frac{1}{2}\cos 20^\circ)(-\frac{1}{2}\cos 40^\circ - \cos 20^\circ)$

$= -\frac{1}{8}\cos 40^\circ - \frac{1}{4}\cos 20^\circ + \frac{1}{4}\cos 40^\circ \cdot \cos 20 + \frac{1}{2}\cos^2 20^\circ$

$L.H.S. = \frac{1}{2}\cdot\frac{1}{2}(2\cos 20^\circ \cdot \cos 40^\circ)\cos 80^\circ$

$= \frac{1}{4}(\cos 60^\circ + \cos 20^\circ) \cdot \cos 80^\circ$

$= \frac{1}{4}\times\frac{1}{2} + \frac{1}{4}(\cos 100^\circ + \cos 60^\circ)$

$= \cancel{\frac{1}{8} - \frac{1}{8} - \frac{1}{4}\cos 100^\circ}$

$= \cancel{-\frac{1}{4}\times \frac{1}{8} + \frac{1}{4}\cos(}$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{4}$

$L.H.S = \frac{1}{2} \cdot 2\cos 20^\circ \cos 40^\circ \cdot \cos 80^\circ \cdot \frac{1}{2}$

$= \frac{1}{4}(\cos 60^\circ + \cos 20^\circ)\cos 80^\circ$

$= \frac{1}{4}\times\frac{1}{2}\cos 80 + \frac{1}{8}(\cos 100 + \cos 60^\circ)$

$\cancel{= \frac{1}{8} + \frac{1}{8}\cos 80^\circ + \frac{1}{4}(90^\circ+10)}$

$\cancel{= \frac{1}{8}\times\frac{1}{2}\times} = \frac{1}{8}\cos 80^\circ + \frac{1}{8}\cos 100^\circ + \frac{1}{8}\times\frac{1}{2}$

$= \frac{1}{8}\cos(90^\circ-10^\circ) + \frac{1}{8}\cos(90^\circ+10^\circ) + \frac{1}{16}$

$= \frac{1}{8}\sin 10^\circ - \frac{1}{8}\sin 10^\circ + \frac{1}{16}$

$= \frac{1}{16} = R.H.S.$

Transformation of sums of diff.

Put $A+B = C$, $A-B = D$

(+) $2A = C+D$ $\therefore A = \frac{C+D}{2}$ (−) $\Rightarrow 2B = C-D$ $\Rightarrow B = \frac{C-D}{2}$

(5) $\Rightarrow \sin C + \sin D = 2\sin\frac{C+D}{2} . \cos\left(\frac{C-D}{2}\right)$

(6) $\Rightarrow \sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

(7) $\Rightarrow \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) . \cos\left(\frac{C-D}{2}\right)$

(8) $\Rightarrow \cos D - \cos C = 2\sin\left(\frac{C+D}{2}\right) . \sin\left(\frac{C-D}{2}\right)$

Date 24.08.2012

Express as Product

(1) $\sin 60 - \sin 80$

$= 2\cos\left(\frac{80}{2}\right) . \sin\left(\frac{20}{2}\right)$

$= 2\cos 40 . \sin 10$

(2) $\cos 10° + \cos 110° + \cos 140°$

$= 2\cos\left(\frac{120°}{2}\right) \cos\left(\frac{100}{2}\right) + \cos 140°$

$= 2\cos 60° . \cos 50° + \cos 140°$

$= \cos 50° + \cos 140°$

$= 2\cos\left(\frac{190}{2}\right) . \cos\left(\frac{90°}{2}\right)$

$= 2\cos 95 \cos 45°$

$= \sqrt{2}\cos 95°$

$= -\sqrt{2}\sin 5°$

(3) $\cos 130° + \cos 110° + \cos 10°$

$= 2\cos\left(\frac{240}{2}\right) \cos\left(\frac{20}{2}\right) + \cos 10°$

$= 2\cos(120°) . \cos(10°) + \cos 10°$

$= -2 \times \frac{1}{2} . \cos(10°) + \cos 10°$

$= 0$

(4) $\cos 20° - \sin 80°$

$= \cos(90°-70°) - \sin 80°$

$= \sin 70° - \sin 80°$

$= 2\cos\left(\frac{70+80°}{2}\right) . \sin\left(\frac{70-80°}{2}\right)$

$= 2\cos 75° \sin(-5°)$

$= -2\sin 5° \cos(90°-15°)$

$= -2\sin 5° . \sin 15°$

(5) $\sin 20 - \cos 50$

$\sin\theta = \cos(90°-\theta)$
$\cos\theta = \sin(90°-\theta)$

$= \cos(90°-20) - \cos 50°$

$= 2\sin\left(\frac{90°-20+50}{2}\right) \sin\left(\frac{50-90+20}{2}\right)$

$= 2\sin\left(\frac{90°+30}{2}\right) . \sin\left(\frac{70-90°}{2}\right)$

(6) P.T.

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right) . \cot\left(\frac{A-B}{2}\right)$$

$$L.H.S. = \frac{\sin A + \sin B}{\sin A - \sin B}$$

$$= \frac{2\sin\left(\frac{A+B}{2}\right) . \cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right) . \sin\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right) . \cot\left(\frac{A-B}{2}\right)$$

$= R.H.S.$

$$\frac{\cos A + \cos B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right) . \cot\left(\frac{A-B}{2}\right)$$

$$L.H.S = \frac{2\cos\left(\frac{A+B}{2}\right) . \cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right) . \sin\left(\frac{A-B}{2}\right)} = \cot\left(\frac{A+B}{2}\right) \cot\left(\frac{A-B}{2}\right)$$

$= R.H.S.$

Q. P.T.

$\sin\theta . \sin(60^\circ-\theta) . \sin(60^\circ+\theta) = \frac{1}{4}\sin 3\theta$.

L.H.S. $= \sin\theta . \sin(60^\circ-\theta) . \sin(60^\circ+\theta)$.

$= \frac{1}{2}\{2\sin(60^\circ+\theta).\sin(60-\theta)\}\sin\theta$

$\cancel{= \frac{1}{2}\{\cos(120^\circ)}$

$= \frac{1}{2}\{\cos 2\theta - \cos 120^\circ\}\sin\theta$

$= \frac{1}{2}\cos 2\theta.\sin\theta - \frac{1}{2}\times(\frac{-1}{2}).\sin\theta.$

$= \frac{1}{2}\cos 2\theta.\cos(90-\theta) + \frac{1}{4}\sin\theta.$

$= \frac{1}{4}\{\cos(90^\circ+\theta) + \cos(90^\circ-3\theta)\} + \frac{1}{4}\sin\theta.$

$= -\frac{1}{4}\cancel{\sin\theta} + \frac{1}{4}\sin 3\theta + \cancel{\frac{1}{4}\sin\theta}$

$= \frac{1}{4}\sin 3\theta \quad = \text{R.H.S}$

Q. $\cos\theta.\cos(60^\circ-\theta).\cos(60^\circ+\theta) = \frac{1}{4}\cos 3\theta$

L.H.S $= \cos\theta.\cos(60^\circ-\theta).\cos(60^\circ+\theta)$

$= \frac{1}{2}\{2\cos(60^\circ-\theta).\cos(60^\circ+\theta)\}\cos\theta$

$= \frac{1}{2}\{\cos(60^\circ-\cancel{\theta}+60^\circ+\cancel{\theta}) + \cos(\cancel{60}-\theta-\cancel{60}-\theta)\}\cos\theta$

$= \frac{1}{2}\{\cos 120^\circ + \cos 2\theta.\cos\theta\}\cos\theta.$

$= -\frac{1}{4}\cos\theta + \frac{1}{4}(2\cos 2\theta\cos\theta).$

$= \cancel{-\frac{1}{4}+\frac{1}{4}}$ $\cos 3\theta = -\frac{1}{4}\cos\theta + \frac{1}{4}.\cos 3\theta + \frac{1}{4}\cos\theta$

$= \frac{1}{4}\cos 3\theta$

$= \text{R.H.S}$

P.T.

$\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$

L.H.S $= \sin 10^\circ.\sin 50^\circ \sin 70^\circ$

$= \frac{1}{2}(2\sin 50^\circ.\sin 70^\circ)\sin 10^\circ$

$= \frac{1}{2}\{\cos(20^\circ) - \cos(120^\circ)\}\sin 10^\circ$

$= \frac{1}{2}\cos 20^\circ.\sin 10 + \frac{1}{4}\sin 10^\circ$

$= \frac{1}{4}\{\sin 30^\circ - \sin 10^\circ\} + \frac{1}{4}\sin 10^\circ$

$= \frac{1}{8} - \cancel{\frac{1}{4}\sin 10^\circ} + \cancel{\frac{1}{4}\sin 10^\circ}$

$= \frac{1}{8} = \text{R.H.S}$

P.T.

$$\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta.$$

L.H.S. $= \dfrac{(\sin\theta + \sin 7\theta) + (\sin 3\theta + \sin 5\theta)}{(\cos\theta + \cos 7\theta) + (\cos 3\theta + \cos 5\theta)}$

$= \dfrac{2.\sin(\frac{8\theta}{2}).\cos(\frac{6\theta}{2}) + 2\sin 4\theta.\cos\theta.}{2\cos 4\theta.\cos 3\theta + 2\cos 4\theta.\cos\theta.}$

$= \dfrac{\sin 4\theta.\cos 3\theta + \sin 4\theta.\cos\theta}{\cos 4\theta.\cos 3\theta + \cos 4\theta.\cos\theta}$

$= \dfrac{\sin 4\theta(\cos 3\theta + \cos\theta)}{\cos 4\theta(\cos 3\theta + \cos\theta)}$

$= \tan 4\theta = \text{R.H.S}$

$$\frac{\sin 2A + \sin 5A - \sin A}{\cos 2A + \cos 5A + \cos A} = \tan 2A$$

L.H.S $= \dfrac{\sin 2A + 2\cos 3A\sin 2A}{\cos 2A + 2\cos 3A\cos 2A} = \tan 2A$

Q. $\sin 2A + \sin 2B + \sin 2C - \sin 2(A+B+C) = 4 \sin(B+C) \sin(C+A) \sin(A+B)$.

$$L.H.S. = \{\sin 2A + \sin 2B\} + \{\sin 2C - \sin 2(A+B+C)\}$$

$$= 2\sin(A+B)\cdot\cos(A-B) + 2\sin\left(\frac{2C - 2A - 2B - 2C}{2}\right)\cos\left(\frac{2C+2A+2B+2C}{2}\right)$$

$$= 2\sin(A+B)\cos(A-B) - 2\sin(A+B)\cos(A+B+2C)$$

$$= 2\sin(A+B)\{\cos(A-B) - \cos(A+B+2C)\}$$

$$= 2\sin(A+B)\left\{2\sin\left(\frac{A-B+A+B+2C}{2}\right) - \sin\left(\frac{A-B-A-B-2C}{2}\right)\right\}$$

$$= 2\sin(A+B)\{2\sin(A+C)\sin(B+C)\}$$

$$= 4\sin(A+B)\sin(B+C)\sin(C+A)$$

$= R.H.S$

P.T. $\cos A + \cos B + \cos C + \cos(A+B+C) = 4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$

$$L.H.S = (\cos A + \cos B) + \{\cos C + \cos(A+B+C)\}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\cos\left\{\frac{C+A+B+C}{2}\right\}\cos\left\{\frac{C-A-B-C}{2}\right\}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 2\cos\left(\frac{A+B+2C}{2}\right)\cos\left(\frac{A+B}{2}\right)$$

$$= 2\cos\left(\frac{A+B}{2}\right)\left\{\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B+2C}{2}\right)\right\}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\frac{\left(\frac{A-B}{2} + \frac{A+B+2C}{2}\right)}{2}\cos\frac{\left(\frac{A-B}{2} - \frac{A+B+2C}{2}\right)}{2}\right\}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\left(\frac{A-B+A+B+2C}{4}\right)\cos\left(\frac{A-B-A-B-2C}{4}\right)\right\}$$

$$= 2\cos\left(\frac{A+B}{2}\right)\left\{2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{B+C}{2}\right)\right\}$$

$$= 4\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{C+A}{2}\right)$$

$= R.H.S$

If $\sin x = k\sin y$, then P.T.

$$\tan\left(\frac{x-y}{2}\right) = \frac{k-1}{k+1}\tan\left(\frac{x+y}{2}\right)$$

$\sin x = k \sin y$

$$\Rightarrow k = \frac{\sin x}{\sin y} \longrightarrow (1)$$

$$\Rightarrow \frac{k-1}{k+1} = \frac{\sin x - \sin y}{\sin y + \sin x}$$

$$\Rightarrow \cancel{\frac{k-1}{k+1}} = \cancel{\frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x-y}{2}\right)\sin\left(\frac{x+y}{2}\right)}}$$

$$\Rightarrow \frac{k-1}{k+1} = \frac{2\sin\left(\frac{x-y}{2}\right)\cdot\cos\left(\frac{x+y}{2}\right)}{2\sin\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{x-y}{2}\right) = \frac{k-1}{k+1}\tan\left(\frac{x+y}{2}\right)$$

Q. If $\sin(\theta+\alpha) = n\sin(\theta-\alpha)$ then P.T. $\cot\theta = \frac{n-1}{n+1}\cot\alpha$

$$n = \frac{\sin(\theta+\alpha)}{\sin(\theta-\alpha)}$$

$$\Rightarrow \frac{n-1}{n+1} = \frac{\sin(\theta+\alpha) - \sin(\theta-\alpha)}{\sin(\theta+\alpha) + \sin(\theta-\alpha)}$$

$$= \frac{2\cos\theta \cdot \sin\alpha}{2\sin\theta \cdot \cos\alpha}$$

$$= \cot\theta \cdot \tan\alpha$$

$$\therefore \cot\theta = \left(\frac{n-1}{n+1}\right)\cot\alpha$$

Q. If $\cos x + \cos y = \frac{1}{3}$, $\sin x + \sin y = \frac{1}{4}$.

P.T. $\tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$

$\cos x + \cos y = \frac{1}{3}$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{1}{3} \longrightarrow (1)$

$\sin x + \sin y = \frac{1}{4}$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right) = \frac{1}{4} \longrightarrow (2)$

$$\frac{(2)}{(1)} \Rightarrow \frac{\cancel{2}\sin\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right) \cdot \cos\left(\frac{x-y}{2}\right)} = \frac{1/4}{1/3}$$

$$\Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{3}{4}$$

Q. If $x\cos\alpha + y\sin\alpha = k = x\cos\beta + y\sin\beta$

then P.T. $\frac{x}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{y}{\sin\left(\frac{\alpha+\beta}{2}\right)} = \frac{k}{\cos\left(\frac{\alpha-\beta}{2}\right)}$

$x(\cos\alpha - \cos\beta) = y(\sin\beta - \sin\alpha) = k$,

~~$\Rightarrow \cos($~~

$\Rightarrow x \cdot 2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \sin\left(\frac{\beta-\alpha}{2}\right) = y\, 2\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$$\Rightarrow \frac{x}{\cos\left(\frac{\alpha+\beta}{2}\right)} = \frac{y}{\sin\left(\frac{\alpha+\beta}{2}\right)}$$

$k = x\cos\beta + y\sin\beta.$

$= \frac{\cos(\alpha+\beta)}{2}$

$= \cos\beta \cdot y \cdot \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)} + y\sin\beta$

$= \frac{y}{\sin\left(\frac{\alpha+\beta}{2}\right)}\left\{\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\beta} + y\sin\beta\right\}$

$= \frac{y}{\sin\left(\frac{\alpha+\beta}{2}\right)}\left\{\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos}\right.$

Q. If $\sin\theta + \sin\phi = a$, $\cos\theta + \cos\phi = b$.

P.T. $\tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

$\sin\theta + \cancel{\cos\theta}\ \sin\phi = a$

$\Rightarrow 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = a$

$\Rightarrow 4\sin^2\left(\frac{\theta+\phi}{2}\right).\cos^2\left(\frac{\theta-\phi}{2}\right) = a^2 \longrightarrow (1)$

$\cos\theta + \cos\phi = b$

$\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right).\cos\left(\frac{\theta-\phi}{2}\right) = b$

$\Rightarrow 4.\cos^2\left(\frac{\theta+\phi}{2}\right).\cos^2\left(\frac{\theta-\phi}{2}\right) = b^2 \longrightarrow (2)$

$\therefore a^2+b^2 = 4\cos^2\left(\frac{\theta-\phi}{2}\right)\left\{\sin^2\left(\frac{\theta+\phi}{2}\right) + \cos^2\left(\frac{\theta+\phi}{2}\right)\right\}$

$= 4\cos^2\left(\frac{\theta-\phi}{2}\right)$

$\therefore \frac{4+(a^2+b^2)}{a^2+b^2} = \frac{4 - 4.\cos^2\left(\frac{\theta-\phi}{2}\right)}{4\cos^2\left(\frac{\theta-\phi}{2}\right)}$

$= \frac{\sin^2\left(\frac{\theta-\phi}{2}\right)}{\cos^2\left(\frac{\theta-\phi}{2}\right)}$

$= \tan^2\left(\frac{\theta-\phi}{2}\right)$

$\therefore \tan\left(\frac{\theta-\phi}{2}\right) = \pm\sqrt{\frac{4-(a^2+b^2)}{a^2+b^2}}$

Q. P.T. $\frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ} = \tan 35^\circ$

L.H.S. $\Rightarrow \frac{1 - \tan 10^\circ}{1 + \tan 10^\circ}$

$= \frac{\tan 45^\circ - \tan 10^\circ}{1 + \tan 45^\circ.\tan 10^\circ}$

$= \tan(45^\circ - 10^\circ)$

$= \tan 35^\circ$

$=$ R.H.S.

MULTIPLE ANGLES :

We know

(1) $\sin(A+B) = \sin A.\cos B + \cos A\sin B$

(2) $\cos(A+B) = \cos A.\cos B - \sin A\sin B$

(3) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

(4) $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot B + \cot A}$

Putting $A = B$ in (1), (2), (3) and (4)

From (1) $\Rightarrow \boxed{\sin 2A = \sin A.\cos A + \cos A.\sin A = 2\sin A\cos A}$

~~(2) $\Rightarrow \cos 2A$~~ $\boxed{= 2\sin A\sqrt{1-\sin^2 A}}$

(2) $\Rightarrow \cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

(3) $\Rightarrow \tan 2A = \frac{2\tan A}{1-\tan^2 A}$

(2) $\Rightarrow \boxed{1 + \cos 2A = 2\cos^2 A}$

and $\boxed{1 - \cos 2A = 2\sin^2 A}$.

(f) $\Rightarrow$ $$\boxed{\cot 2A = \frac{\cot^2 A - 1}{2\cot A}}$$

Q. Find $\sin 3A$, $\cos 3A$, $\tan 3A$ and $\cot 3A$ in terms of there respective ratios

(i) $\sin(3A) = \sin(2A + A)$

$= \sin 2A . \cos A + \cos 2A . \sin A$

$= 2\sin A . \cos A . \cos A + (\cos^2 A - \sin^2 A) \sin A$

$= 2 . \sin A \cos^2 A + \sin A . (1 - 2\sin^2 A)$

$= 2 \sin \cos^2 A : \cancel{+} \sin A - 2\sin^3 A$

$= 2 \sin A (1 - \sin^2 A) + \sin A - 2\sin^3 A$

$= 2\sin A + \sin A - 4\sin^3 A$

$= 3\sin A - 4\sin^3 A$

(ii) $\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin A . \sin 2A$

$= (\cancel{1}\,2\cos^2 A)\cos A - \sin A . 2\sin A . \cos A$

$= -\cos A + 2\cos^3 A - 2\cos A\,(1 - \cos^2 A)$

$= -\cos A + 2\cos^3 A - 2\cos A + 2\cos^3 A$

$$\boxed{\cos 3A = 4\cos^3 A - 3\cos A}$$

(iii) $$\tan 3A = \frac{\tan 2A + \tan A}{1 - \tan A . \tan 2A}$$

$$= \frac{\frac{2\tan A}{1 - \tan^2 A} + \tan A}{1 - \tan A \frac{2\tan A}{1 - \tan^2 A}}$$

$$\Rightarrow \frac{2\tan A + \tan A - \tan^3 A}{1 - \tan^2 A - 2\tan^2 A}$$

$$\boxed{\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}}$$

$$\cot 3A = \frac{1}{\tan 3A}$$

$$= \frac{\cot A \cot 2A - 1}{\cot 2A + \cot A}$$

$$\Rightarrow \frac{\cot A . \frac{\cos^2 A - 1}{2\cot A} - 1}{\frac{\cos^2 A - 1}{2\cot A} + \cot A}$$

$$\Rightarrow \frac{\cot^3 A - \cot A - 2\cot A}{\cot^2 A - 1 + 2\cot^2 A}$$

$$\boxed{\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}}$$

Express $\sin 2A$ and $\cos 2A$ in term of $\tan A$

We know

$\sin 2A = 2\sin A \cos A$

$= 2 \frac{\sin A}{\cos A} . \cos^2 A$

$= 2\tan A . \frac{1}{\sec^2 A}$

$$\boxed{\sin 2A = \frac{2\tan A}{1 + \tan^2 A}}$$

$\cos 2A = 2\cos^2 A - 1$

$= \frac{2}{\sec^2 A} - 1 = \frac{2}{1 + \tan^2 A} - 1$

$= (\cos^2\theta - \sin^2\theta)\{3\sin^2\theta.\cos^2\theta + (\cos^2\theta - \sin^2\theta)^2\}$

$= (\cos^2\theta - \sin^2\theta)\{(\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta\}$

$= (\cos^2\theta - \sin^2\theta)\{(\cos^2\theta + \sin^2\theta)^2\}$

$= (\cos^2\theta - \sin^2\theta)^3 + 3\cos 2\theta.\sin^2\theta(\cos^2\theta - \sin^2\theta)$

$= (2\cos^2\theta - 1)^3 + 3\cos^2\theta\sin^2\theta.\cos 2\theta$

$= \cos^3 2\theta + 3\cos^2\theta.\sin^2\theta.\cos 2\theta.$

$= \cos 2\theta(\cos^2 2\theta + 3\cos^2\theta.\sin^2\theta.)$

$= \cos 2\theta(\cos^4\theta + \sin^4\theta + \cos^2\theta.\sin^2\theta)$

$= \cos 2\theta\{(\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\cos^2\theta.\sin^2\theta - \cos^2\theta\sin^2\theta\}$

$= \cos 2\theta\{(\cos^2\theta + \sin^2\theta)^2 - 4.\cos^2\theta\sin^2\theta.\frac{1}{4}\}$

$= \cos 2\theta\{1^2 - \sin^2 2\theta.\frac{1}{4}\}$

$= \cos 2\theta(1 - \frac{1}{4}\sin^2 2\theta)$

$=$ R.H.S.

Prove that

(1) $\dfrac{\sin 2A}{1+\cos 2A} = \tan A$.

L.H.S $= \dfrac{\sin 2A}{1+\cos 2A}$

$= \dfrac{2\sin A\cos A}{2\cos^2 A}$

$= \dfrac{2\sin A}{2\cos A}$

$= \tan A$

$=$ R.H.S

$\dfrac{\sin 2A}{1-\cos 2A} = \cot A.$

L.H.S. $= \dfrac{\sin 2A}{1-\cos 2A}$

$= \dfrac{2\sin A\cos A}{2\sin^2 A}$

$= \cot A$

$\cot A - \tan A = 2\cot 2A$.

L.H.S. $= \cot A - \tan A$

$= \dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\cos A}$

$= \dfrac{\cos^2 A - \sin^2 A}{\sin A\cos A}$

$= \dfrac{2\cos 2A}{\sin 2A}$

$= 2\cot 2A$

$=$ R.H.S

(4) $(2\cos\theta+1)(2\cos\theta-1) = 2\cos 2\theta+1$

L.H.S. $= (2\cos\theta+1)(2\cos\theta-1)$

$= 4\cos^2\theta - 1$

$= 2(2\cos^2\theta) - 1$

$= 2(1+\cos 2\theta) - 1$

$= 2\cos 2\theta + 1$

$=$ R.H.S.

(5). $\tan\theta\{1+\sec 2\theta\} = \tan 2\theta$

L.H.S $= \tan\theta\{1+\sec 2\theta\}$

$= \tan\theta\left\{1+\frac{1}{\cos 2\theta}\right\}$

$= \tan\theta\left\{\frac{1+\cos 2\theta}{\cos 2\theta}\right\}$

$= \tan\theta \, \frac{2\cos^2\theta}{\cos 2\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}\cdot 2\cos^2\theta}{\cos 2\theta}$

$= \frac{2\sin\theta\cos\theta}{\cos 2\theta} = \frac{\sin 2\theta}{\cos 2\theta}$

$= \tan 2\theta$

$=$ R.H.S.

(6) $\frac{\cot A - \tan A}{\cot A + \tan A} = \cos 2A$.

L.S.H. $= \frac{\cot A - \tan A}{\cot A + \tan A} = \frac{\cos A/\sin A - \sin A/\cos A}{\cos A/\sin A - \sin A/\cos A}$

$= \frac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$

$= \cos 2A$

$=$ R.H.S

(7) $\tan A + \cot A = 2\operatorname{cosec} 2A$

L.H.S $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$= \frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}$

$= \frac{1\times 2}{2\sin A\cos A}$

$= 2\operatorname{cosec} 2A$

(8) $\cos^4\theta - \sin^4\theta = \cos 2\theta$.

L.H.S. $= (\cos^2\theta)^2 - (\sin^2\theta)^2$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$= 1.\cos 2\theta = \cos 2\theta =$ R.H.S

9. $\cos^6\theta + \sin^6\theta = \frac{1}{4}[1+3\cos^2 2\theta]$

L.H.S $= (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta+\sin^2\theta)^3 - 3.\cos^2\theta\sin^2\theta(\sin^2\theta+\cos^2\theta)$

$= 1 - 3\cos^2\theta\sin^2\theta$.

$= 1 - \frac{1}{4}.3.4\cos^2\theta.\sin^2\theta$.

$= 1 - \frac{3}{4}(\sin 2\theta)^2$

$= 1 - \frac{3}{4}(1-\cos^2 2\theta)$

$= 1 - \frac{3}{4} + \frac{3}{4}\cos^2\theta = \frac{1}{4}(1+3\cos^2 2\theta) =$ R.H.S

(10) $\dfrac{\sin^2\alpha - \sin^2\beta}{\sin\alpha\cos\alpha - \sin\beta\cos\beta} = \tan(\alpha+\beta)$

$$= \frac{\sin(\alpha+\beta)\sin(\alpha-\beta)}{\cancel{\sin(\alpha-\beta)}\ \frac{1}{2}\sin 2\alpha - \frac{1}{2}\sin 2\beta}$$

$$= \frac{2\sin(\alpha+\beta)\sin(\alpha-\beta)}{\sin 2\alpha - \sin 2\beta}$$

$$= \frac{\cancel{2}\sin(\alpha+\beta)\ \cancel{\sin(\alpha-\beta)}}{\cancel{2}\ \cancel{\sin\left(\frac{2\alpha-2\beta}{2}\right)}\cos(\alpha+\beta)}$$

$= \tan(\alpha+\beta)$

$= R.H.S$

(11) $\dfrac{1-\cos 2\theta + \sin 2\theta}{1+\cos 2\theta + \sin 2\theta} = \tan\theta$

$$\frac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sin\theta\,(\sin\theta + \cos\theta)}{\cancel{2}\cos\theta\,(\cos\theta + \sin\theta)}$$

$= \tan\theta = R.H.S$

(12) $\dfrac{\sin\alpha - \sqrt{1+\sin 2\alpha}}{\cos\alpha - \sqrt{1+\sin 2\alpha}} = \cot\alpha$ (α being acute and +ve)

$$= \frac{\sin\alpha - \sqrt{(\sin\alpha + \cos\alpha)^2}}{\cos\alpha - \sqrt{(\sin\alpha + \cos\alpha)^2}}$$

$$= \frac{\sin\alpha - \sin\alpha - \cos\alpha}{\cos\alpha - \sin\alpha - \cos\alpha}$$

$= \cot\alpha = R.H.S$

(13) $\dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} - \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = 2\tan 2\theta$

$$L.H.S = \frac{(\cos\theta + \sin\theta)^2 - (\cos\theta - \sin\theta)^2}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2.2.\cos\theta\sin\theta}{\cos 2\theta}$$

$$= \frac{2\sin 2\theta}{\cos 2\theta}$$

$= 2\tan 2\theta = R.HS$

(14) $\dfrac{\sin 4\theta}{\cos 2\theta} \times \dfrac{1-\cos 2\theta}{1-\cos 4\theta} = \tan\theta$

$$= \frac{2\sin 2\theta.\cos 2\theta}{(\cos^2\theta - \sin^2\theta)} \times \frac{2\sin^2\theta}{2\sin^2 2\theta}$$

$$= \frac{2.2\sin^2\theta}{2.2\sin\theta.\cos\theta}$$

$= \tan\theta = R.H.S$

(15) $\dfrac{\cos A - \sin A}{\cos A + \sin A} = \sec 2A - \tan 2A$

$$L.H.S = \frac{(\cos A - \sin A)(\cos A - \sin A)}{(\cos A + \sin A)(\cos A - \sin A)}$$

3

$$\frac{\cos^2 A - \sin^2 A}{\sin^2 A + \cos^2 A \pm 2\sin A\cos A}$$

$$= \frac{\cancel{\cos 2A}}{\cancel{1 + \sin 2A}}$$

$$= \frac{1 \pm \sin 2A}{\cos 2A}$$

$$= \sec 2A - \tan 2A$$

(1) $$\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} = 1 - \frac{1}{2}\sin 2\theta.$$

(2) $$\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A.$$

(3) $\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \cdot \tan 2\theta \tan\theta.$

(4) $$\frac{\cot A}{\cot A - \cot 3A} - \frac{\tan A}{\tan 3A - \tan A} = 1$$

(5) $\sin 8\theta = 8\sin\theta \cdot \cos\theta \cdot \cos 2\theta \cdot \cos 4\theta.$

(6) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$

Q. If α and β are acute angle and $\cos 2\alpha = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$, then P.T.

$\tan\alpha = \sqrt{2}\tan\beta$

$$\frac{\cos 2\alpha}{1} = \frac{3\cos 2\beta - 1}{3 - \cos 2\beta}$$

$$\Rightarrow \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha} = \frac{(3 - \cos 2\beta) - (3\cos 2\beta - 1)}{(3 - \cos 2\beta) + (3\cos 2\beta - 1)}$$

$$\Rightarrow \frac{2\sin^2\alpha}{2\cos^2\alpha} = \frac{4 - 4\cos 2\beta}{2 + 2\cos 2\beta}$$

$$\therefore \tan^2\alpha = \frac{4(1 - \cos 2\beta)}{2(1 + \cos 2\beta)}$$

$$= \frac{4 \cdot 2\sin^2\beta}{2 \cdot 2\cos^2\beta}$$

$\tan^2\alpha = 2\tan^2\beta$

$\therefore \tan\alpha = \sqrt{2}\tan\beta$

Sub multiple angle

(i) $\sin 2A = 2\sin A \cdot \cos A$

(ii) $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$1 + \cos 2A = 2\cos^2 A$, $1 - \cos 2A = 2\sin^2 A$

(iii) $$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

(iv) $$\cot 2A = \frac{\cot^2 A - 1}{2\cot A}$$

(5) $\sin 3A$

Put $2A = \theta$ $\Rightarrow A = \theta/2$; $3A = \theta$ $\Rightarrow A = \theta/3$

(i) $\sin\theta = 2\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}$

(ii) $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$

$1 + \cos\theta = 2\cos^2\frac{\theta}{2}$; $1 - \cos\theta = 2\sin^2\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1}{2}(1 + \cos\theta)}$, $\sin\frac{\theta}{2} = \sqrt{\frac{1}{2}(1 - \cos\theta)}$

(iii) $$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$$

(iv) $$\cot\theta = \frac{\cot^2\frac{\theta}{2} - 1}{2\cot\frac{\theta}{2}}$$

(12) (5) $\sin\theta = 3\sin\frac{\theta}{3} - 4\sin^3\frac{\theta}{3}$

(6) $\cos\theta = 4\cos^3\frac{\theta}{3} - 3\cos\frac{\theta}{3}$

For $3A$

(7) $\tan\theta = \dfrac{3\tan\frac{\theta}{3} - \tan^3\frac{\theta}{3}}{1 - 3\tan^2\frac{\theta}{3}}$

Q. ~~Express $1 \pm \sin\theta$ is a perfect sq.~~

$1 + \sin\theta$.

$= \left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right) \pm \left(2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}\right)$

$(1 \pm \sin\theta) = \left(\sin\frac{\theta}{2} \pm \cos\frac{\theta}{2}\right)^2$

$\sqrt{1 \pm \sin\theta} = \sin\frac{\theta}{2} \pm \cos\frac{\theta}{2}$

Q. Express $\sin\theta$ and $\cos\theta$ in terms of $\tan\frac{\theta}{2}$.

* $\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$

$= \dfrac{2\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\cdot\cos^2\frac{\theta}{2}$

$= \dfrac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

* $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$= \dfrac{2}{1 + \tan^2\frac{\theta}{2}} - 1$

$= \dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

Q. Find all T-ratios of 18°, 36°, 54° and 72°

(i) Let $\theta = 18^\circ$

$\Rightarrow 5\theta = 90^\circ$

$\Rightarrow 3\theta + 2\theta = 90^\circ$

$\Rightarrow 2\theta = 90^\circ - 3\theta$

$\Rightarrow \sin 2\theta = \sin(90^\circ - 3\theta)$

$\Rightarrow \sin 2\theta = \cos 3\theta$.

$\Rightarrow 2\sin\theta\cdot\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 2\sin\theta = 4\cos^2\theta - 3$.

$\Rightarrow 4(1 - \sin^2\theta) - 2\sin\theta - 3 = 0$

$\Rightarrow 4 - 4\sin^2\theta - 2\sin\theta - 3 = 0$

$\Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \dfrac{-2 \pm \sqrt{2^2 - 4\cdot 4\cdot(-1)}}{2\cdot 4}$

$= \dfrac{-2 \pm \sqrt{20}}{8}$

$= \dfrac{-2 \pm 2\sqrt{5}}{8}$

$= \dfrac{-1 \pm \sqrt{5}}{4}$

$\sin 18^\circ = \cancel{\dfrac{\sqrt{5}+(-1)}{4}}\ \dfrac{\sqrt{5}-1}{4}$ $\quad\because \theta$ is in 1st quadrant

$\therefore \cos 18^\circ = \sqrt{1 - \left(\dfrac{\sqrt{5}-1}{4}\right)^2}$

$= \sqrt{\dfrac{16 - 5 - 1 + 2\sqrt{5}}{16}}$

$= \sqrt{\dfrac{10 + 2\sqrt{5}}{16}} = \dfrac{\sqrt{10 + 2\sqrt{5}}}{4}$

~~$\tan 18^\circ$~~ $= \dfrac{\frac{\sqrt{5}-1}{4}}{\frac{\sqrt{5+\sqrt{5}}}{2}}\cdot\frac{2}{2}$ $= \dfrac{\sqrt{5}-1}{\sqrt{5+\sqrt{5}}}\cdot\dfrac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2}(\sqrt{5}-1)}{2\sqrt{5+\sqrt{5}}}$

$\tan 18^\circ = \dfrac{\frac{\sqrt5 - 1}{\cancel{4}}}{\frac{\sqrt{10+2\sqrt5}}{\cancel{4}}}$

$= \dfrac{\sqrt5 - 1}{\sqrt{10+2\sqrt5}}$ //

Sin 18°

(ii) $\theta = 36^\circ$

~~$\Rightarrow 5\theta = 180^\circ$~~

~~$\Rightarrow 3\theta = 180^\circ - 2\theta$~~

~~$\Rightarrow \sin 3\theta = \cos(180^\circ - 2\theta)$~~

$\Rightarrow \theta = 2\times 18^\circ$

We know.

$2\cos^2 \theta/2 = 1 + \cos\theta.$

$\Rightarrow 2\cos^2 \frac{36}{2} = 1 + \cos 36^\circ$

$\Rightarrow 2\cos^2 18^\circ = 1 + \cos 36^\circ$

$\Rightarrow \cos 36^\circ = 2\times\left(\dfrac{\sqrt{10+2\sqrt5}}{4}\right)^2 - 1$

$= \cancel{2}\times \dfrac{10+2\sqrt5}{\cancel{16}\,8} - 1$

$= \dfrac{2\sqrt5 + 2}{8}$

$= \dfrac{\sqrt5 + 1}{4}$ //

$\sin 36^\circ = \sqrt{1 - \left(\dfrac{\sqrt5+1}{4}\right)^2}$

$= \dfrac{\sqrt{10-2\sqrt5}}{4}$ //

(iii) $\theta = 54^\circ$

$= 90^\circ - 36^\circ$

$\Rightarrow \sin\theta = \sin(90^\circ - 36^\circ)$

$\Rightarrow \sin 54^\circ = \cos 36^\circ$

$= \dfrac{\sqrt5+1}{4}$

$\cos 54^\circ = \dfrac{1}{4}\left(\sqrt{10-2\sqrt5}\right)$ //

(iv). $\theta = 72^\circ$

$= 90^\circ - 18^\circ$

$\Rightarrow \sin 72^\circ = \cos 18^\circ$

$= \dfrac{\sqrt{10+2\sqrt5}}{4}$

$\cos 72^\circ = \dfrac{\sqrt5 - 1}{4}$ //

Q. P.T

$\dfrac{1-\cos A}{\sin A} = \tan A/2$

L.H.S $= \dfrac{1-\cos A}{\sin A}$

$= \dfrac{2\sin^2 A/2}{2\sin A/2 \cdot \cos A/2}$

$= \tan A/2$

$=$ R.H.S //

Q. $\dfrac{1+\cos A}{\sin A} = \cot A/2$ //

$= \dfrac{2\cos^2 A/2}{2\sin A/2 \cos A/2}$

$= \cot A/2$ //

Q.

$$\left(\sin \tfrac{A}{2} \pm \cos \tfrac{A}{2}\right)^2 = 1 \pm \sin A.$$

L.H.S. $= \left(\sin A/_2 \pm \cos A/_2\right)^2$

$= \left(\sin^2 A/_2 + \cos^2 A/_2\right) \pm 2\sin A/_2 . \cos A/_2$

$= 1 \pm \sin A$

$=$ R.H.S

Q. $\sec\theta + \tan\theta = \tan\left(\pi/_4 + \theta/_2\right)$.

L.H.S $=$ $\sec\theta + \tan\theta$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$= \cancel{\frac{\tan \pi/_4 + 2\sin\theta/_2 . \cos\theta/_2}{2\cos\ldots 1 - 2\sin^2\theta/_2}}$$

$$= \frac{1+\sin\theta}{\cos^2\theta/_2 - \sin^2\theta/_2}$$

$$= \frac{\left(\sin\theta/_2 + \cos\theta/_2\right)^2}{\cos^2\theta/_2 - \sin^2\theta/_2}$$

$$= \frac{\sin\theta/_2 + \cos\theta/_2}{\cos\theta/_2 - \sin\theta/_2}$$

$$= \frac{1+\tan\theta/_2}{1-\tan\theta/_2}$$

$= \tan\left(\pi/_4 + \theta/_2\right)$

Q. ~~$\sec\theta + \tan\theta$~~

$$\frac{1+\sin\theta - \cos\theta}{1+\sin\theta+\cos\theta} = \tan\theta/_2$$

$$= \frac{2\sin^2\theta/_2 + 2\sin\theta/_2\cos\theta/_2}{2\cos^2\theta/_2 + 2\sin\theta/_2\cos\theta/_2} = \frac{2\sin\theta/_2\left(\sin\theta/_2 + \cos\theta/_2\right)}{2\cos\theta/_2\left(\cos\theta/_2 + \sin\theta/_2\right)}$$

$= \tan\theta/_2$

Q. $$\frac{1+\sin x}{1-\sin x} = \tan^2\left(\pi/_4 + x/_2\right)$$

$$\Rightarrow \frac{\left(\sin x/_2 + \cos x/_2\right)^2}{\left(\sin x/_2 - \cos x/_2\right)^2}$$

$$\Rightarrow \left[\frac{1+\tan x/_2}{\tan x/_2 - 1}\right]^2$$

$$= \left[\frac{\tan \pi/_4 + \tan x/_2}{-\left(\tan \pi/_4 - \tan\pi/_4 . \tan x/_2\right)}\right]^2$$

$= \left[\tan\left(\pi/_4 + x/_2\right)\right]^2$

$= \tan^2\left(\pi/_4 + x/_2\right)$

Q. $$\frac{\sin\alpha/_2 - \sqrt{1+\sin\alpha}}{\cos\alpha/_2 - \sqrt{1+\sin\alpha}} = \cot\alpha/_2 \qquad 0 < \alpha < \pi$$

$$\Rightarrow \frac{\sin\alpha/_2 - \left(\sin\alpha/_2 + \cos\alpha/_2\right)}{\cos\alpha/_2 - \left(\sin\alpha/_2 + \cos\alpha/_2\right)}$$

$$= \frac{-\cos\alpha/_2}{-\sin\alpha/_2} \qquad \because 0 < \alpha < \pi \quad \therefore 0 < \alpha/_2 < \pi/_2$$

$= \cot\alpha/_2$

Q $$\frac{\sin 2\theta}{1+\cos 2\theta} . \frac{\cos\theta}{1+\cos\theta} = \tan\theta/_2$$

$$= \frac{2\sin\theta . \cos\theta}{2\cos^2\theta} . \frac{\cos\theta}{2\cos^2\theta/_2}$$

$$= \frac{2\sin\theta/_2 . \cos\theta/_2}{2\cos^2\theta/_2}$$

$= \tan\theta/_2$

Q. Find the value of $\sin 22\frac{1}{2}^\circ$ and $\cos 22\frac{1}{2}^\circ$.

Let $\theta = 45^\circ$

$\Rightarrow \theta/2 = 22\frac{1}{2}^\circ$

$\Rightarrow \cos\theta = \cos 45$

$\Rightarrow 1 - 2\sin^2\theta/2 = 1/\sqrt{2}$

$\Rightarrow 2\sin^2\theta/2 = 1 - 1/\sqrt{2}$

$\Rightarrow \sin^2\theta/2 = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$\Rightarrow \sin\theta/2 = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2\sqrt{2}}}$

$\therefore \sin 22\frac{1}{2}^\circ = \sqrt{\frac{1(\sqrt{2}-1)}{\cancel{\sqrt{2}}\ \sqrt{2}}}$

$\cos 22\frac{1}{2}^\circ = \sqrt{1 - \frac{2+1-2\sqrt{2}}{4\times 2}}$

$= \sqrt{\frac{8-3+2\sqrt{2}}{8}}$

$= \sqrt{\frac{5+2\sqrt{2}}{8}}$

$= \sqrt{\frac{(2)^2+1^2+2\sqrt{2}}{4.\sqrt{2}.\sqrt{2}}}$

P.T.

(1) $\frac{1+\tan A/2}{1-\tan A/2} = \frac{1+\sin A}{\cos A}$

(2) $2\cos \pi/16 = \sqrt{2+\sqrt{2+\sqrt{2}}}$

(3) If $\tan\phi/2 = \sqrt{\frac{1-e}{1+e}}\ \tan\theta/2$ then show that $\cos\phi = \frac{\cos\theta - e}{1-e\cos\theta}$

(3) $\tan\phi/2 = \sqrt{\frac{1+e}{1-e}}\ \tan\theta/2$

We know

$\cos\phi = \frac{1-\tan^2\phi/2}{1+\tan^2\phi/2}$

$= \frac{1-\left(\frac{1+e}{1-e}\right)\tan^2\theta/2}{1+\left(\frac{1+e}{1-e}\right)\tan^2\theta/2}$

$= \frac{1-e-\tan^2\theta/2 - e\tan^2\theta/2}{1-e+\tan^2\theta/2 + e\tan^2\theta/2}$

$= \frac{(1-\tan^2\theta/2) - e(1+\tan^2\theta/2)}{(1+\tan^2\theta/2) - e(1-\tan^2\theta/2)}$

$= \frac{\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} - e}{1 - e.\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}}$

$= \frac{\cos\theta - e}{1-e\cos\theta}$

TRIGONOMETRIC IDENTITIES

If A, B, C be angles of a ΔABC then

$A+B+C = \pi \Rightarrow$

$A+B = \pi - C \Rightarrow \sin(A+B) = \sin(\pi - C) = \sin C$

$B+A = \pi - B \Rightarrow \cos(A+B) = \cos(\pi - C) = -\cos C$

$\tan(A+B) = \tan(\pi - C) = -\tan C$

$\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$.

$A+B+C = \pi$

$\Rightarrow A+B = \pi - C \longrightarrow (1)$

and $C = \pi - (A+B)$. $\longrightarrow (2)$

L.H.S $= (\sin 2A + \sin 2B) + \sin 2C$

$= 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(\pi - C)\cdot\cos(A-B) + 2\sin C\cos C$

$\Rightarrow 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A-B) + \cos C\}$

$= 2\sin C\{\cos(A-B) + \cos\{\pi - (A+B)\}\}$

$= 2\sin C\{\cos(A-B) - \cos(A+B)\}$

$= 2\sin C \cdot 2\sin A \sin B$

$= 4\sin A\sin B\cdot\sin C$

Q. $\sin A + \sin B + \sin C = 4\cos A/_2 \cdot \cos B/_2 \cos C/_2$

$A+B+C = \pi$

$\Rightarrow A/_2 + B/_2 + C/_2 = \pi/_2$

$\Rightarrow C/_2 = \pi/_2 - (A/_2 + B/_2) \longrightarrow (1)$

$(A/_2 + B/_2) = (\pi/_2 - C/_2) \longrightarrow (2)$

L.H.S $= \sin A + \sin B + \sin C$

$= 2\sin(A+B/_2)\cos(A-B/_2) + 2\sin C/_2\cos C/_2$

$= 2\sin\{\pi/_2 - C/_2\}\cos\left(\frac{A-B}{2}\right) + 2\sin C/_2\cdot\cos C/_2$

$= 2\sin C/_2\cos(A-B/_2) + 2\sin C/_2\cdot\cos C/_2$

$= 2\sin C/_2\left\{\cos\left(\frac{A-B}{2}\right) + \cos\left\{\pi/_2 - \left(\frac{A+B}{2}\right)\right\}\right\}$

$= 2\sin C/_2\left[\cos\left(\frac{A-B}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right]$

$= \cancel{2\sin C/_2 \cdot 2\sin A/_2}$

$= 2\cos C/_2\; 2\cos A/_2 \cdot \cos B/_2$

Q. $A+B+C = \pi$, P.T.

$\cos A + \cos B + \cos C = 1 + 4\sin A/_2 \cdot \sin B/_2 \cdot \sin C/_2$

$A+B+C = \pi$

$\Rightarrow \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \pi/_2$

$\Rightarrow C/_2 = (\pi/_2 - A+B/_2) \longrightarrow (1)$

$\Rightarrow A+B/_2 = \pi/_2 - C/_2 \longrightarrow (2)$

L.H.S $= \cos A + \cos B + \cos C$

$= 2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right) + \cos C$

$= 2\cos(\pi/_2 - C/_2)\cdot\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2 C/_2$

$= 2\sin C/_2\cos\left(\frac{A-B}{2}\right) - 2\sin\left\{\pi/_2 - \frac{A+B}{2}\right\}\sin C/_2 + 1$

$= 2\sin C/_2\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\} + 1$

$= 1 + 2\sin C/_2\{2\sin(A/_2)\sin(B/_2)\}$

$= 1 + 4\sin A/_2\cdot\sin B/_2\cdot\sin C/_2$

$=$ R.H.S

$A+B+C = \pi/_2$. P.T.

$\tan A \tan B + \tan C \cdot \tan A = 1$

$A+B+C = \pi/_2$

$\Rightarrow \tan(A+B+C) = \tan\pi/_2$

$\Rightarrow \frac{\tan(A+B) + \tan C}{1 - \tan(A+B)\tan C} = \infty$

$\therefore 1 - \tan(A+B)\tan C = 0$

$\Rightarrow 1 - \frac{(\tan A + \tan B)}{1 - \tan A\tan B}\cdot\tan C = 0$

$\Rightarrow 1-\tan A\tan B-(\tan A+\tan B)\tan C=0$

$\Rightarrow \tan A+\tan B+\tan$

$\Rightarrow \tan A\tan B+\tan B\cdot\tan C+\tan C\tan A=1$

Q. If $A+B+C=\pi/2$ P.T.

$\sin A+\sin B-\sin C=4\sin A/2\,\sin B/2\,\sin C/2$

$A+B+C=\pi$

$\Rightarrow \frac{A+B}{2}=\pi/2-C/2 \longrightarrow (1)$

$C/2=\pi/2-(A+B)/2 \longrightarrow (2)$

L.H.S. $=\sin A+\sin B-\sin C$

$=2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)-2\sin$

$=\cancel{2\sin\{\pi/2-C/2\}\cos\left(\frac{A-B}{2}\right)-2\sin C/2}$

$\cancel{2\cos C\cdot\cos\left(\frac{A-B}{2}\right)}\ldots$

$=2\sin\{\pi/2-C/2\}\cos\left(\frac{A-B}{2}\right)-2\sin C/2\cos C/2$

$=2\cos C/2\left\{\cos\frac{A-B}{2}-\sin C/2\right\}$

$=2\cos C/2\left\{\cos\frac{A-B}{2}-\cos\left(\frac{A+B}{2}\right)\right\}$

$=4\cos C/2\cdot\sin A/2\,\sin B/2$

Q. In any $\triangle ABC$ P.T.

$\tan A/2\tan B/2+\tan B/2\cdot\tan C/2+\tan C/2\tan A/2=0$

$A+B+C=\pi$

$\Rightarrow \frac{A+B+C}{2}=\frac{\pi}{2}$

$\Rightarrow \frac{A+B}{2}=\frac{\pi}{2}-C/2$

$\Rightarrow \tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan(\pi/2-C/2)$

$\Rightarrow \frac{\tan A/2+\tan B/2}{1-\tan A/2\cdot\tan B/2}=\cot C/2$

$\Rightarrow \frac{\tan A/2+\tan B/2}{1-\tan A/2\cdot\tan B/2}=\frac{1}{\tan C/2}$

$\Rightarrow \tan A/2\cdot\tan C/2+\tan C/2\cdot\tan B/2=1-\tan A/2\cdot\tan B/2$

$\Rightarrow \tan A/2\cdot\tan B/2+\tan B/2\tan C/2+\tan C/2\tan A/2=1$

Q. $A+B+C=\pi$, P.T.

$\cos^2 A+\cos^2 B+\cos^2 C+2\cos A\cdot\cos B\cos C=1$

$\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cdot\cos B\cos C$

$\Rightarrow \frac{1}{2}(1+\cos 2A)+\frac{1}{2}(1+\cos 2B)+\cancel{\frac{1}{2}(1+\cos 2C)}\cos^2 C$

$=\frac{1}{2}(2+\cos 2A+\cos 2B+\cancel{\cos 2C})+\cos^2 C$

$=\frac{1}{2}\{2+2\cos(A+B)\cdot\cos(A-B)\}+\cos^2 C$

$=1+\cos(A+B)\cos(A-B)+\cos^2 C$

$=1+\cos\{\pi-C\}\cos(A-B)+\cos C\cdot\cos\{\pi-(A+B)\}$

$=1-\cos C\{\cos(A-B)+\cos(A+B)\}$ Mind Here

$=1-\cos C\cdot 2\cdot\cos A\cos B$

$=1-2\cos A\cdot\cos B\cos C$

INVERSE TRIGONOMETRIC FUNCTION :-

If $\sin\theta = x$, then θ is called the sine inverse of x and it is written $\sin^{-1}x$.

i.e. when $\sin\theta = x \longrightarrow (1)$

$\Rightarrow \theta = \sin^{-1}x \longrightarrow (2)$

From (1) & (2)

$$\sin(\sin^{-1}x) = x$$
$$\cos(\cos^{-1}x) = x$$
$$\tan(\tan^{-1}x) = x$$

$$\boxed{\begin{aligned}\theta &= \sin^{-1}(\sin\theta)\\ \theta &= \cos^{-1}(\cos\theta)\\ \theta &= \tan^{-1}(\tan\theta)\end{aligned}}$$

(2) If $\sin\theta = x$ then $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{x}$

$\theta = \sin^{-1}x$ $\qquad \theta = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

$$\boxed{\begin{aligned}\sin^{-1}x &= \operatorname{cosec}^{-1}(1/x)\\ \tan^{-1}x &= \cot^{-1}(1/x)\\ \cos^{-1}x &= \sec^{-1}\left(\tfrac{1}{x}\right)\end{aligned}}$$

(3) If $\sin\theta = x$ then $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$

$\theta = \sin^{-1}x$ $\qquad \theta = \cos^{-1}\sqrt{1-x^2}$

$$\boxed{\sin^{-1}x = \cos^{-1}\sqrt{1-x^2} = \sec^{-1}\frac{1}{\sqrt{1-x^2}}}$$

(4) $\sin\theta = x$, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1-x^2}}$

$\theta = \sin^{-1}(x)$ $\qquad \theta = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

$$\boxed{\begin{aligned}\sin^{-1}(x) &= \tan^{-1}\frac{x}{\sqrt{1-x^2}}\\ &= \cot^{-1}\frac{\sqrt{1-x^2}}{x}\end{aligned}}$$

(5) If $\sin\theta = x$ then $\sin\theta = \cos(\pi/2 - \theta)$ ~~$\Rightarrow \sin\theta$~~

$\theta = \sin^{-1}x$, ~~$\theta = \sin^{-1}\{\cos(\pi/2-\theta)\}$~~

~~$\pi/2 - \theta = \cos^{-1}$~~ $\Rightarrow \cos(\pi/2 - \theta) = x$

$\Rightarrow \pi/2 - \theta = \cos^{-1}x$

$\Rightarrow \pi/2 = \theta + \cos^{-1}x$

$$\boxed{\begin{aligned}\pi/2 &= \sin^{-1}x + \cos^{-1}x.\\ \pi/2 &= \tan^{-1}x + \cot^{-1}x\\ \pi/2 &= \sec^{-1}x + \operatorname{cosec}^{-1}x\end{aligned}}$$

Standard formulae on Inverse circular function

(i) $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$

(ii) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left\{xy \mp \sqrt{(1-x^2)(1-y^2)}\right\}$

(iii) $\tan^{-1}x \pm \tan^{-1}y = \tan^{-1}\left\{\frac{x \pm y}{1 \mp xy}\right\}$

(iv) $\cot^{-1}x \pm \cot^{-1}y = \cot^{-1}\left\{\frac{xy \mp 1}{y \pm x}\right\}$

P.T.

(i) $2\sin^{-1}x = \sin^{-1}\left\{2x\sqrt{1-x^2}\right\}$

(ii) $2\cos^{-1}x = \cos^{-1}\{2x^2-1\}$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}$

(iv) $2\cot^{-1}x = \cot^{-1}\frac{x^2-1}{2x}$

(v) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}$

(vi) $2\tan^{-1}x = \cot^{-1}\frac{1-x^2}{1+x}$.

$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}$

$y = x$

$\sin^{-1}x + \sin^{-1}x = \sin^{-1}\left\{x\sqrt{1-x^2} + x\sqrt{1-x^2}\right\}$

$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

(ii) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left\{ x y - \sqrt{1-x^2}\sqrt{1-y^2} \right\}$

$\cos^{-1}x + \cos^{-1}x = \cos^{-1}\left\{ x^2 - \sqrt{1-x^2}\sqrt{1-x^2} \right\}$

$\Rightarrow 2\cos^{-1}x = \cos^{-1}(x^2 - 1 + x^2\}$

$\Rightarrow 2\cos^{-1}x = \cos^{-1}(2x^2-1)$

(iii). $\tan^{-1}x = A$

$\Rightarrow x = \tan A$.

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$\Rightarrow 2\tan^{-1}x = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

(iv) $\cot^{-1}x = A$.

$\Rightarrow \cot A = x$.

$\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$.

$\Rightarrow 2A = \cot^{-1}\left(\dfrac{x^2-1}{2x}\right)$

$\Rightarrow 2\cos^{-1}x = \cot^{-1}\left(\dfrac{x^2-1}{2x}\right)$

(v) ~~Sin2A~~ $\tan^{-1}x = A$

$\Rightarrow \tan A = x$

$\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

$\Rightarrow 2A = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$\Rightarrow 2.\tan A = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

(vi) $\tan^{-1}\cancel{x} = A$

$\Rightarrow \cancel{x} = \tan A$

$\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

$\Rightarrow 2.\tan^{-1}(x) = \cos\left(\dfrac{1-x^2}{1+x^2}\right)$

P.T.

(i) $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

(ii) $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$

(iii) $3\tan^{-1}x = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$

(iv) $3\cot^{-1}x = \cot^{-1}\left(\dfrac{3x^2-1}{x^3-3x}\right)$

(i) Let $\sin^{-1}x = A \Rightarrow \sin A = x$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\Rightarrow 3A = \sin^{-1}(3\sin A - 4\sin^3 A)$

$\Rightarrow 3\sin^{-1}x = \sin^{-1}(3x-4x^3)$.

(ii) $\cos^{-1}x = A \rightarrow \cos A = x$.

$\cos 3A = 4\cos^3 A - 3\cos A$

$\Rightarrow 3A = \cos(4x^3-3x)$

$\Rightarrow 3\cos^{-1}x = \cos(4x^3-3x)$

(iii) $\tan^{-1}x = A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$\Rightarrow 3\tan^{-1}x = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$

(iv) $\cot^{-1}x = A$

$\cot 3A = \dfrac{3\cot^2 A - 1}{\cot^3 A - 3\cot A}$

$\Rightarrow 3\cot^{-1}x = \cot^{-1}\left(\dfrac{3x^2-1}{x^3-3x}\right)$

Q. P.T. $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$.

Let $2\tan^{-1}\frac{1}{3} = x$

L.H.S. $= \tan^{-1}\left(\frac{2/3}{1-(1/3)^2}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{2/3}{8/9}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{3/4 + 1/7}{1 - 3/4 \cdot 1/7}\right)$

$= \tan^{-1}\left(\frac{21+4}{28-3}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right)$

$= \tan^{-1} 1 = \frac{\pi}{4}$

(ii) $\tan^{-1}\frac{1}{7} + \cos^{-1}\frac{4}{5} = \frac{\pi}{4}$

L.H.S. $= \tan^{-1}\frac{1}{7} + \cos^{-1}\frac{4}{5}$

$= \tan^{-1}\frac{1}{7} + \tan^{-1}\left(\frac{\sqrt{1-(4/5)^2}}{4/5}\right)$

$= \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{1/7 + 3/4}{1 - 1/7 \cdot 3/4}\right)$

$= \tan^{-1}(1) = \frac{\pi}{4}$

Q. $2\tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\frac{12}{5}$

$2\tan^{-1}(2/3) = \tan^{-1}\left\{\frac{2 \cdot 2/3}{1-(2/3)^2}\right\}$

$= \tan^{-1}\left\{\frac{4/3}{5/9}\right\} = \tan^{-1}\left(\frac{12}{5}\right)$

Q. Solve

$\tan^{-1}x + \tan 3x = \cot^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \tan^{-1}\frac{x+3x}{1-x \cdot 3x} = \tan^{-1}2$

$\Rightarrow \frac{4x}{1-3x^2} = 2$

$\Rightarrow 2 - 6x^2 = 4x$

$\Rightarrow 3x^2 + 2x - 1 = 0$

$\Rightarrow 3x^2 + 3x - x - 1 = 0$

$\Rightarrow 3x(x+1) - 1(x+1) = 0$

$x = 1/3$ or $x = -1$

Q. Find the value of

$\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$

$= \sin\frac{\pi}{2}$ $\quad \because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$= 1$

Q. $\tan(\tan^{-1}a + \cot a)$

$= \tan\frac{\pi}{2}$

$= \infty$

Q. $\tan\left\{\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right\}$

$= \tan\left\{\frac{1}{2} \cdot 2\tan^{-1}x + \frac{1}{2} \cdot 2\tan^{-1}y\right\}$

$= \tan . \frac{\pi}{2} = \infty$

P.T.

$\tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-x^2}$

L.H.S. $= \tan^{-1}x + \tan^{-1}\frac{2x}{1-x^2}$

$= \tan^{-1}x + 2\tan^{-1}x$

$= 3\tan^{-1}x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$

8. $\tan^{-1}\frac{3}{5} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$

L.H.S. $= \tan^{-1}\frac{3}{5} + \sin^{-1}\frac{3}{5}$

$= \tan\frac{3}{5} + \tan^{-1}\frac{3/5}{\sqrt{1-(3/5)^2}}$

$= \tan^{-1}\frac{3/5 + \cancel{3/5}/\cancel{4/5}}{1-3/5\cdot 3/4}$

$= \tan^{-1}\frac{\frac{12+15}{20}}{\frac{20-9}{20}}$

$= \tan^{-1}\left(\frac{27}{11}\right)$

Quadratic equation and its expression

α and β are the roots of $ax^2+bx+c=0$ then

$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$ $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

(1) Sum of the roots

$\alpha+\beta = -b/a = \frac{-\text{Coeff of } x}{\text{Coeff of } x^2}$

(2) Product of the roots

$\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coeff of } x^2}$

(3) Roots are reciprocal of.

$\alpha\beta = 1$

$c/a = 1 \Rightarrow c=a$

(4) Roots are equal in magnitude but opposite in sign.

$\alpha+\beta = 0$

$\Rightarrow -b/a = 0 \Rightarrow b=0$

re of character of the roots of $ax^2+bx+c=0$

re of the roots depend on the discriminant $\sqrt{b^2-4ac}$

hen $b^2-4ac=0$, then the roots are equal and real

" $b^2-4ac>0$, " " " " real and distinct

" $b^2-4ac<0$, " " " " imaginary and unequal.

when $b^2-4ac \geq 0$ then the roots are real

" b^2-4ac is a perfect square then roots are rational.

(0 perfect square তুলি হবা নহয়)

tion of a Quadratic eq^n:-

α,β are roots of $ax^2+bx+c=0$ → (1) then

have $\alpha+\beta = -b/a \Rightarrow \frac{b}{a} = -(\alpha+\beta)$.

$\alpha\beta = c/a \Rightarrow \frac{c}{a} = \alpha\beta$.

Divide eq^n ① by a

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 + \{-(\alpha+\beta)\}x + \alpha\beta = 0.$$

$$\Rightarrow \boxed{x^2 - (\alpha+\beta)x + \alpha\beta = 0}$$

$$\Rightarrow \boxed{x^2 - \{\text{Sum of the roots}\}x + \text{product of the roots} = 0}$$

* If one root is $(m+\sqrt{n})$ then its conjugate $m-\sqrt{n}$ is the other root

Eg-① If α and β are the roots of $2x^2-3x+1=0$, find the value of ① $\alpha^2+\beta^2$. ② $(\alpha-\beta).(\alpha+\beta)$ ③ $\frac{1}{\alpha^3}+\frac{1}{\beta^3}$

α, β roots of $2x^2-3x+1=0$

$$\alpha+\beta = \frac{3}{2} \rightarrow ① \quad \alpha\beta = \frac{1}{2} \rightarrow ⑪$$

① $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$$= \left(\frac{3}{2}\right)^2 - 2\times\frac{1}{2}$$

$$= \frac{9}{4} - 1 = \frac{5}{4}$$

(ii) $(\alpha-\beta)(\alpha+\beta) = (\alpha^2-\beta^2)$

$$= \alpha -$$

$\cancel{\alpha = \frac{3}{2} - \beta}$ ⑪ $\Rightarrow \cancel{\left(\frac{3}{2}-\beta\right)\beta = \frac{1}{2}}$

$$\Rightarrow \cancel{\frac{(3-2\beta)\beta}{2} = \frac{1}{2}}$$

$$\Rightarrow 3\beta - 2\beta^2 = 1$$

$$\Rightarrow 2\beta^2 - 3\beta + 1 = 0$$

$$\Rightarrow 2\beta^2 - 2\beta - \beta -$$

$$\alpha-\beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$$

$$= \sqrt{\left(\frac{3}{2}\right)^2 - 4\times\frac{1}{2}} = \sqrt{\frac{9-8}{4}} = \frac{1}{2}$$

(ii) $(\alpha-\beta)(\alpha+\beta) = \frac{1}{2}\times\frac{3}{2} = \frac{3}{4}$

$\alpha+\beta = \frac{3}{2}$, $\alpha - \beta = \frac{1}{2}$

$2\alpha = 2 \qquad 2\beta = 1$

$\Rightarrow \alpha = 1, \qquad \Rightarrow \beta = \frac{1}{2}$

∴ (iii) $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{1}{1^3} + \frac{1}{(\frac{1}{2})^3}$

$$= 1 + 8 = 9$$

$$\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$= \frac{27}{8} - \frac{3}{2}\times$$

Q. If the roots of the quadratic eq^n $x^2+px+q=0$ are in the ratio 1:4 show that $4p^2 = 25q$

Soln:-

Let the roots are α and 4α

∴ $\alpha + 4\alpha = -p \qquad \alpha.4\alpha = q$

$\Rightarrow 5\alpha = -\frac{p}{5} \qquad \Rightarrow \alpha^2 = \frac{q}{4}$

$$\Rightarrow \alpha^2 = \frac{p^2}{25}$$

$$∴ \frac{p^2}{25} = \frac{q}{4}$$

$$\Rightarrow 4p^2 = 25q$$

Q. If k be the ratio of the roots of $ax^2+bx+c=0$ show that $\frac{(k+1)^2}{k} = \frac{b^2}{ac}$

Let the roots are α and $k\alpha$

$$∴ \cancel{\frac{\alpha}{\beta} = k}$$

$$\cancel{\alpha = k\beta}$$

$\alpha + k\alpha = -\frac{b}{a} \qquad \alpha k\alpha = \frac{c}{a}$

$\Rightarrow \alpha(1+k) = -\frac{b}{a} \qquad \Rightarrow k\alpha^2 = \frac{c}{a}$

$$\Rightarrow \alpha = \frac{-b}{a(1+k)}$$

$$\Rightarrow k \times \frac{b^2}{a^2(1+k)^2} = \frac{c}{a}$$

$$\Rightarrow \frac{(1+k)^2}{k} = \frac{ab^2}{a^2c}$$

$$= \frac{b^2}{ac}$$

Q. If α and β are the roots of $ax^2+bx+c=0$, find the eqn whose roots are $1/\alpha$, $1/\beta$

$\because$ α, β are roots of eqn $ax^2+bx+c=0$

$\alpha+\beta = -b/a$, $\alpha\beta = c/a$

$x^2-(\alpha+\beta)x+\alpha\beta=0$

The reqd quadratic eqn whose roots $1/\alpha$ and $1/\beta$

$$x^2-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x+\frac{1}{\alpha}\cdot\frac{1}{\beta}=0$$

$$\Rightarrow x^2-\left(\frac{\alpha+\beta}{\alpha\beta}\right)x+\frac{1}{\alpha\beta}=0$$

$$\Rightarrow x^2-\frac{-b/a}{c/a}x+\frac{1}{c/a}=0$$

$$\Rightarrow x^2+\frac{b}{c}x+\frac{a}{c}=0$$

$$\Rightarrow cx^2+bx+a=0$$

Q. If P and Q are the roots of $3x^2+6x+2=0$ Find the eqn where roots are $-p^2/q$ and $-q^2/p$.

P and Q are roots of $3x^2+6x+2=0$

$\therefore P+Q=-2$, $PQ=2/3$

The quadratic eqn whose roots are $-p^2/q$ and $-q^2/p$ is

$$x^2-\left\{\left(-p^2/q+-q^2/p\right)\right\}x+\frac{-p^2}{q}\cdot\frac{-q^2}{p}=0$$

$$\Rightarrow x^2+\left(\frac{p^3+q^3}{PQ}\right)x+PQ=0$$

$$\Rightarrow PQx^2+\{(P+Q)^3-3PQ(P+Q)\}x+(PQ)^2=0$$

$$\Rightarrow \frac{2}{3}x^2+\left\{(-2)^3-3\cdot\frac{2}{3}\cdot(-2)\right\}x+\left(\frac{2}{3}\right)^2=0$$

$$\Rightarrow \frac{2}{3}x^2\{-8+4\}x+\frac{4}{9}=0$$

$$\Rightarrow \frac{2x^2}{3}-4x+\frac{4}{9}=0$$

$$\Rightarrow 18x^2-36x+4=0$$

$$\Rightarrow 3x^2-18x+2=0$$

Q. If $(2+3i)$ is one of the roots of a qua. eqn., find the reqd quadratic eqn.

Ans $(2+3i)$ is one root $\therefore$ the other roots is $(2-3i)$

$\therefore$ Reqd quadratic eqn.

$$x^2-\{(2+3i)+(2-3i)\}x+(2+3i)(2-3i)=0$$

$$\Rightarrow x^2-(4)x+(4+9)=0$$

$$\Rightarrow x^2-4x+13=0$$

Determinant of 3rd order

Let the 3 linear eqs

$a_1x+b_1y+c_1=0 \longrightarrow (1)$

$a_2x+b_2y+c_2=0 \longrightarrow (2)$

$a_3x+b_3y+c_3=0 \longrightarrow (3)$

Solving (2) and (3) by the method of cross multiplication

$$\frac{x}{b_2c_3-c_2b_3}=\frac{y}{c_2a_3-a_2c_3}=\frac{1}{a_2b_3-a_3b_2}$$

$$x=\frac{b_2c_3-c_2b_3}{a_2b_3-a_3b_2}, \quad y=\frac{c_2a_3-a_2c_3}{a_2b_3-a_3b_2}$$

~~Solving (2) and (3) by the method~~

Putting x and y in (1)

$$a_1\left\{\frac{b_2c_3-b_3c_2}{a_2b_3-a_3b_2}\right\}+b_1\left\{\frac{c_2a_3-c_3a_2}{a_2b_3-a_3b_2}\right\}+c_1=0$$

$$\Rightarrow a_1(b_2c_3-b_3c_2)+b_1(c_2a_3-c_3a_2)+c_1(a_2b_3-a_3b_2)=0$$

$$\Rightarrow a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)=0 \quad (4)$$

Eq^n (4) is the general solution of eq^n (1), (2) and (3) whose L.H.S. can be written

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \Delta$$

→ Rows R_1

Δ (Delta) is known as the determinant of 3rd order where $a_1, b_1, c_1, \ldots$ are the elements of the determinant.

3×3 → no. of rows, no. of columns

Column ↓ c_1 c_2 c_3

no. of Rows = columns

Expansion of the determinant (Rowwise)

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$= a_1(b_2c_3-b_3c_2)-b_1(a_2c_3-a_3c_2)+c_1(a_2b_3-a_3b_2)$$

Columnwise Expansion :

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_2 & c_3 \end{vmatrix} - a_2\begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

Ex:- Evaluate

$$\begin{vmatrix} 2 & -3 & 1 \\ -1 & 2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} - (-3)\begin{vmatrix} -1 & 3 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix}$$

$$= 2(2+6)+3(-1-9)+1(2-6)$$

$$= 16-30-4$$

$$= -34+16 = -18$$

(2) $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ where ω is the imaginary cube root of unit

$$= 1(\omega^2\cdot\omega-1)-\omega(\omega^2-\omega^2)+\omega^2(\omega-\omega^4)$$

$$= 1(\omega^3-1)-0+\omega^2(\omega-\omega^3\omega) \qquad \because \omega^3=1$$

$$= 0-0+\omega^2\cdot 0 = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = 1(1-6)-1(3+2)+1(4+1)$$

$$= -5-\cancel{5}+\cancel{5} = -5$$

$$\begin{vmatrix} 3 & 1 & 4 \\ 5 & 1 & 3 \\ 1 & -3 & -4 \end{vmatrix} = 3(-4+9)-1(3+20)+4(-15-1)$$

$$= 3\times5-23-60$$

$$= 15-83 = -68$$

Properties of determinant :-

(1) In a determinant if the rows are change into columns and columns into rows, then the value of the determinant is not change.

$$\begin{vmatrix} 3 & 4 \\ -5 & 1 \end{vmatrix} \xrightarrow[R_2\leftrightarrow c_2]{R_1\leftrightarrow c_1} \begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix}$$

$3+20 = 3-(-20)$

$23 = 23$

(ii) In a determinant if two adjacent rows and column are interchange then the numerical value of the det^n is the same but the sign is changed.

(iii) $\begin{vmatrix} 7 & 1 \\ 2 & -3 \end{vmatrix} \overset{R_1 \leftrightarrow R_2}{=} - \begin{vmatrix} 2 & -3 \\ 7 & 1 \end{vmatrix}$

$\Rightarrow -21-2 = -(2+21)$

$\Rightarrow -23 = -(23)$

(iii) In a determinant if any two rows or columns are identical then the value of the determinant is zero

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & -7 \\ 1 & 2 & 3 \end{vmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} = 0 \quad [\because R_1 \text{ and } R_3 \text{ are same}]$$

(iv) In a determinant if every elements of any row or column is multiplied by the same term then the det^n can be written as the multiplication by that term.

$$\begin{vmatrix} ma_1 & nb_1 & c_1 \\ ma_2 & nb_2 & c_2 \\ ma_3 & nb_3 & c_3 \end{vmatrix} = n.m \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

(v) In a determinant if every element of any row or column consist of sum of two term then the determinant can be written as the sum of two determinant of the same order

eg
$$\begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$$

Q Using the properties of determinant evaluate

(i) $\begin{vmatrix} 99 & 81 & 321 \\ 88 & 72 & 423 \\ 55 & 45 & 657 \end{vmatrix}$

$$= \begin{vmatrix} 9\times 11 & 9\times 9 & 312 \\ 8\times 11 & 8\times 9 & 423 \\ 5\times 11 & 9\times 5 & 657 \end{vmatrix} = 11\times 9 \begin{vmatrix} 9 & 9 & 312 \\ 8 & 8 & 423 \\ 5 & 5 & 657 \end{vmatrix}$$

$= 0 \quad [C_1 = C_2]$

Q. $\begin{vmatrix} 14 & 18 & 21 \\ 15 & 19 & 22 \\ 16 & 20 & 23 \end{vmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} = \begin{vmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ 16 & 20 & 23 \end{vmatrix} = 0$

Applying $R_1 = R_1 - R_2$

$R_2 = R_2 - R_3$

Q $\begin{vmatrix} 63 & 72 & 81 \\ 100 & 109 & 97 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 9\times 7 & 9\times 8 & 9\times 9 \\ 100 & 109 & 97 \\ 7 & 8 & 9 \end{vmatrix}$

$= 9 \begin{vmatrix} 7 & 8 & 9 \\ 100 & 109 & 97 \\ 7 & 8 & 9 \end{vmatrix} = 0$

P.T.

Q. $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$

$\because 1+\omega+\omega^2 = 0$

Applying $C_1 = C_1 + C_2 + C_3$

$$\begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ \omega+\omega^2+1 & \omega^2 & 1 \\ \omega^2+1+\omega & 1 & \omega \end{vmatrix} = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = 0$$

Q. P.T.

$$\begin{vmatrix} p & q & r \\ p^2 & q^2 & r^2 \\ p^3 & q^3 & r^3 \end{vmatrix} = pqr\,(p-q)(q-r)(r-p)$$

~~$p(q^2r^3 - r^2q^3) - q(p^2r^3 - p^3r^2) + r(p^2q^3 - p^3q^2)$~~

$$\text{LHS} = pqr \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ p^2 & q^2 & r^2 \end{vmatrix}$$

applying $C_1 = C_1 - C_2$, $C_2 = C_2 - C_3$

$$= pqr \begin{vmatrix} 0 & 0 & 1 \\ p-q & q-r & r \\ p^2-q^2 & q^2-r^2 & r^2 \end{vmatrix}$$

$$\cancel{= pqr \{(p-q)(q^2-r^2) - (p-r)(p^2-q^2)\}}$$

$$= pqr(p-q)(q-r) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & r \\ (p+q) & q+r & r^2 \end{vmatrix}$$

$$= pqr(p-q)(q-r)\{(q+r) - (p+q)\}$$

$$= pqr(p-q)(q-r)(r-p)$$

Solve the linear eqn by Cramers Rule

Let the 3 linear eqn be

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Let Δ, Δ_1, Δ_2 and Δ_3 be the determinants of 3rd order where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Then by Cramers Rule.

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

$(\Delta \neq 0)$

Solve by Cramers Rule

$3x - 2y - 5 = 0$

$x + 2y + 3 = 0$

$3x - 2y = 5$

$x + 2y = -3$

Let Δ, Δ_1 and Δ_2 be the determinants of 2nd order, where

$$\Delta = \begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix} = 6+2 = 8 \neq 0$$

$$\Delta_1 = \begin{vmatrix} 5 & -2 \\ -3 & 2 \end{vmatrix} = 10 - 6 = 4$$

$$\Delta_2 = \begin{vmatrix} 3 & 5 \\ 1 & -3 \end{vmatrix} = -9-5 = -14$$

Then by Cramers Rule.

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z$$

$$= \frac{4}{8} = \frac{1}{2}, \quad y = \frac{\cancel{14}^{\,7}}{\cancel{8}_{\,4}} = -\frac{7}{4}$$

(2)

$x + y + z = 3$

$2x - y + 3z = 4$

$x + 2y - z = 2$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = 1(1-6) - 1(-3-2) + 1(4+1) = -5-5+5 = -5$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 3 \\ 2 & -1 & 4 \\ 1 & 2 & 2 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & 1 \\ 4 & -1 & 3 \\ 2 & 2 & -1 \end{vmatrix} = 3(1-$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 3 \\ 1 & 2 & -1 \end{vmatrix} =$$

$x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$, $z = \frac{\Delta_3}{\Delta}$

$= \frac{5}{5}$ $\quad = \frac{6}{5}$ $\quad = \frac{5}{5}$

$= 1$ $\quad = 1$ $\quad = 1$

(i) $3x + y + 4z = 0$
$5x + y + 3z = 1$
$x - 3y - 4z = 5$

(ii) $x + 2y - 4z = 3$
$3x - 4y + 5z = 7$
$7x - y + z = 14$

(iii) $x + y + z = 1$
$x + 2y + z = 2$
$x + y + 2z = 0$

$$\Delta = \begin{vmatrix} 3 & 1 & 4 \\ 5 & 1 & 3 \\ 1 & -3 & -4 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 4 \\ -1 & 1 & 3 \\ -5 & -3 & -4 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} 3 & 0 \\ 5 & -1 \\ 1 & -5 \end{vmatrix}$$

Exponial and Logarithmic Series

(1) Factorial Notation :- The product of n natural nos to no is denoted by using $\lfloor n$ or $n!$

i.e. $n! = 1.2.3. \ldots n$
$= n(n-1)(n-2)(n-3) \ldots 3.2.1.$

$3! = 3.2.1$ $\quad 0! = 1$
$3! = 3\cdot 2!$
$1! = n\,(n-1)!$ $\quad (-n)! = \infty$
$(n-1)! = (n-1)\,(n-2)!$

(2) Exponential Series :-

An expansion of the form
$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots + \infty$$
is known as exponential series and it is written by as e^x

i.e. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \infty \to$ (i)

Putting $x = -x$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \infty \to \text{(ii)}$$

P.T. (i) $e^x + e^{-x} = 2\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \infty\right]$.

(ii) $e^x - e^{-x} = 2\left[\frac{x}{1!} + \frac{x^3}{3!} + \ldots + \infty\right]$

(i) + (ii) $\Rightarrow e^x + e^{-x} = \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \infty\right) + \left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\right)$

$$= 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right.$$

(1) − (ii) ⇒ $e^x - e^{-x} = \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \infty\right)$

$- \left(1 - \frac{x}{1!} + \frac{x^2}{2!} + \cdots \infty\right)$

$= 2\left(\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right)$

$$\Rightarrow \boxed{\frac{1}{2}(e^x - e^{-x}) = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}$$

Putting $x = 1$ in (i)

$e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty$

$\Rightarrow e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty \rightarrow (3)$

Putting $x = -1$ in (i)

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots \infty \rightarrow (4)$

(3)+(4) ⇒

$e^1 + e^{-1} = \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \infty\right) + \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \cdots \infty\right)$

$= 2\left(1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots \infty\right)$

$$\Rightarrow \boxed{\frac{1}{2}(e^1 + e^{-1}) = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots}$$

(3) − (4) ⇒

$\frac{1}{2}(e^1 - e^{-1}) = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} \cdots \infty$

Logarithmic Series: The expansion of the form

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots \quad (|x| < 1)$

is known as logarithmic series and it is known $\log_e(1+x)$.

i.e. $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \rightarrow (1)$

Putting $x = -x$ in (1)

$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$

$= -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\right)$

$\Rightarrow -\log_e(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$ (2)

Q. P.T. (i) $\log \frac{1+x}{1-x} = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right]$

(ii) $\log \frac{1}{\sqrt{1-x^2}} = \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$

Show that

(1) $\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right) = 1$

L.H.S. $= e^x \times e^{-x}$

$= e^0$

$= 1$

(2) $\left(1 + \frac{1}{1!} + \frac{1}{2!} + \cdots \infty\right)\left(1 - \frac{1}{1!} + \frac{1}{2!} + \cdots \infty\right)$

$= e^1 \times e^{-1}$

$= 1$

(3) $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots \infty = \frac{1}{e}$

$= \frac{3-1}{3!} + \frac{5-1}{5!} + \frac{7-1}{7!} + \cdots \infty$

$= \frac{3}{3!} - \frac{1}{3!} + \frac{5}{5!} - \frac{1}{5!} + \frac{7}{7!} - \frac{1}{7!} + \cdots$

$= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \frac{1}{7!} \cdots$

$= \left(1 - \frac{1}{1!}\right) + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \cdots$

$= e^{-1}$

(4) Show that

$$\frac{2}{1!}+\frac{4}{3!}+\frac{6}{5!}+\dots\infty = e$$

$$=\frac{1+1}{1!}+\frac{3+1}{3!}+\frac{5+1}{5!}+\dots\infty$$

$$=\frac{1}{1!}+\frac{1}{1!}+\frac{3}{3!}+\frac{1}{3!}+\frac{5}{5!}+\frac{1}{5!}+\dots\infty$$

$$=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\frac{1}{5!}+\dots\infty$$

$$=e^1$$

(5) $$\left(1+\frac{1}{2!}+\frac{1}{4!}+\dots\infty\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+\dots\right)^2=1$$

$$=\left\{\tfrac12(e^1+e^{-1})\right\}^2-\left\{\tfrac12(e^1-e^{-1})\right\}^2$$

$$=\frac14\left\{\cancel{e^2}+\cancel{e^{-2}}+2-\cancel{e^2}-\cancel{e^{-2}}+2\right\}$$

$$=\frac14\times4=1$$

(6) $$\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\dots}{\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\dots}=\frac{e-1}{e+1}$$

L.H.S. $$=\frac{\left(1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}\dots\right)-1}{\left(1+\frac{1}{1!}+\frac{1}{3!}+\frac{1}{5!}+\dots\right)-1}$$

$$=\frac{\frac12(e+e^{-1})-1}{\frac12(e-e^{-1})}$$

$$=\frac{\frac12\left(e^2+\frac{1}{e}\right)}{\frac12\left(e^2-\frac{1}{e}\right)}$$

$$=\frac{e^2+1-2e}{e^2-1}=\frac{(e-1)^2}{(e-1)(e+1)}$$

Q. $$\log 2=\frac{1}{1.2}+\frac{1}{3.4}+\frac{4}{5.6}+\dots$$

$$\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{2}-\frac{x^4}{4}+\dots$$

Putting $x=1$

$$\Rightarrow \log_e(1+1)=1-\frac12+\frac13-\frac14+\dots$$

$$\Rightarrow \log 2=\left(1-\tfrac12\right)+\left(\tfrac13-\tfrac14\right)+\left(\tfrac15-\tfrac16\right)+\dots$$

$$\therefore \log 2=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\dots$$ Proved

Q. P.T.

$$\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\dots=1-\log 2$$

~~1-log 2~~

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$$

$x=1$ put করি

$$\Rightarrow \log(1+1)=\left(1-\frac12\right)+\left(\frac13-\frac14\right)+\dots$$

$$\Rightarrow \cancel{\log 2=1}$$

$$\Rightarrow 1-\log 2=\left(\frac12-\frac13\right)+\left(\frac14-\frac15\right)+\dots$$

$$1-\log 2=\frac{1}{2.3}+\frac{1}{4.5}+\dots$$ Proved

Q. If $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$ $|x|<1$

Show that $x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+\dots\infty$

$$y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}\dots$$

$$\Rightarrow \log_e(1+x) \qquad y=\log_e(1+x)$$

$$\Rightarrow e^y=1+x$$

$$\Rightarrow \cancel{1}+\frac{y}{1!}+\frac{y^2}{2!}+\frac{y^3}{3!}+\dots$$

$\therefore x = y + \frac{y^2}{\lfloor 2} + \frac{y^3}{\lfloor 3} + \frac{y^4}{\lfloor 4} + \cdots$

5. $1 + \frac{3}{\lfloor 1} + \frac{5}{\lfloor 2} + \frac{7}{\lfloor 3} + \cdots = 3e$

L.H.S $\frac{1}{\lfloor 0} + \frac{3}{\lfloor 1} + \frac{5}{\lfloor 2} + \cdots$

$T_n = a + (n-1)d$

Here $T_n = \frac{1+(n-1)2}{\lfloor 0 + (n-1)1}$

$= \frac{2n-2+1}{\lfloor n-1}$

$= \frac{2(n-1)+1}{\lfloor n-1}$

$= \frac{2(n-1)}{(n-1)\lfloor n-2} + \frac{1}{\lfloor n-1}$

$T_n = \frac{2}{\lfloor n-2} + \frac{1}{\lfloor n-1} \longrightarrow (1)$

Putting $n = 1, 2, 3, \ldots$ (1)

$T_1 = 2 \cdot \frac{1}{\lfloor 1-2} + \frac{1}{\lfloor 0} = 2 \cdot \frac{1}{\lfloor -1} + \frac{1}{1} = 2.0 + 1$

$T_2 = 2 \cdot \frac{1}{\lfloor 2-2} + \frac{1}{\lfloor 2-1} = 2 \cdot \frac{1}{1} + \frac{1}{\lfloor 1}$

$T_3 = 2 \cdot \frac{1}{\lfloor 3-2} + \frac{1}{\lfloor 3-1} = 2 \cdot \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2}$

$T_4 = 2 \cdot \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3}$

$\ldots \ldots$

$\therefore T_1 + T_2 + T_3 + \ldots = 2\left[1 + \frac{1}{\lfloor 2} + \cdots\right] + \left[1 + 1/\lfloor 1 + 1/\lfloor 2 + 1/\lfloor 3 \cdots\right]$

$= 2e + e = 3e$

6. $\frac{1.3}{\lfloor 1} + \frac{2.4}{\lfloor 2} + \frac{3.5}{\lfloor 3} + \cdots = 4e$

$T_n = \frac{n \cdot \{3 + (n-1)1\}}{\lfloor 1 + (n-1)1}$

$= \frac{n(3+n-1)}{\lfloor n}$

$= \frac{n(n+2)}{\lfloor n}$

$= \frac{n+2}{\lfloor n-1} = \frac{(n-1)+3}{\lfloor n-1}$

$T_1 = \frac{(1-1)+3}{\lfloor 1-1} = \frac{0+3}{\lfloor 0} = 0 + \frac{3}{\lfloor 0}$

$T_2 = \frac{(2-1)+3}{\lfloor 2-1} = \frac{1}{\lfloor 1} + \frac{3}{\lfloor 1}$

$T_3 = \frac{(3-1)+3}{\lfloor 3-1} = \frac{2}{\lfloor 2} + \frac{3}{\lfloor 2}$

$T_1 + T_2 + T_3 + \ldots = \left(0 + \frac{1}{\lfloor 1} + \frac{2}{\lfloor 2} + \frac{3}{\lfloor 3} + \cdots\right) + 3\left(1 + 1/\lfloor 1 + 1/\lfloor 2 + 1/\lfloor 3 + \cdots\right)$

$=$

$= 3e$

Q. $\frac{1.2}{\lfloor 1} + \frac{2.3}{\lfloor 2} + \frac{3.4}{\lfloor 3} + \dots\dots$

$$T_n = \frac{n\{2+(n-1)1\}}{\lfloor n}$$

$$= \frac{n+1}{\lfloor n-1}$$

$$= \frac{(n-1)+2}{\lfloor n-1}$$

$$= \frac{1}{\lfloor n-2} + \frac{2}{\lfloor n-1}$$

$$T_1 = \frac{1}{\lfloor 1-2} + \frac{2}{\lfloor 1-1} = 0 + \frac{2}{1}$$

$$T_2 = \frac{1}{\lfloor 0} + \frac{2}{\lfloor 1} = \frac{1}{1} + \frac{2}{\lfloor 1}$$

$$T_3 = \frac{1}{\lfloor 1} + \frac{2}{\lfloor 2} = 1/\lfloor 1 + \frac{2}{\lfloor 2}$$

$$T_4 = \frac{1}{\lfloor 2} + \frac{2}{\lfloor 3}$$

$$T_1 + T_2 + T_3 + T_4 + \dots\dots = \left(1 + 1/\lfloor 1 + 1/\lfloor 2 + \dots \infty\right) + 2\left(\frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \dots \infty\right)$$

$$= e + 2e$$

$$= \underline{\underline{3e}}$$

Q. $1 + \frac{1+2}{\lfloor 2} + \frac{1+2+3}{\lfloor 3} + \frac{1+2+3+4}{\lfloor 4} + \dots\dots = \frac{3}{2}e$

Here $T_n = \frac{1+2+3+\dots\dots+n}{\lfloor n}$

$$= \frac{n(n+1)/2}{\lfloor n}$$

$$= \frac{n(n+1)}{2n\lfloor n-1}$$

$$= \frac{(n-1)+2}{2\lfloor n-1} = \frac{1}{2\lfloor n-2} + \frac{2}{2\lfloor n-1}$$

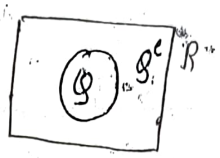
$$= \frac{1}{2\lfloor n}$$

COMPLEX NUM...

MATHS Ist Sem

1. Natural no, $N \in \{1, 2, 3, \ldots\}$
2. Integer, $I \in \{0, \pm 1, \pm 2, \ldots\}$
3. Rational no, $Q \in \{ p/q ; p, q \in I, q \neq 0\}$
4. Irrational, $Q^c \in$ which are not rational $(\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots)$

$R = Q \cup Q^c$



1. Solve

$x^2 + 9 = 0$

$\Rightarrow x = 3i$

$\sqrt{-a} \times \sqrt{-a} = -a$

$(\sqrt{-a})^2 = -a$

So, this new system of numbers ± 3 we say imaginary numbers (or impossible numbers)

The boundary of number system is extended to include this new class of numbers called imaginary or impossible numbers.

$i = \sqrt{-1}$ is called fundamental imaginary unit.

$i^2 = -1$

$i^3 = i^2 . i = -i$

$i^4 = i^2 . i^2 = -1 . -1 = 1$

$i^5 = i^4 . i = i$

$i^6 = i^4 . i^2 = -1$

$i^7 = i^4 . i^3 = -i$

$i^8 = 1$

$i^9 = i$

$i^{10} = -1$

$i^{3n+1} = i^{3n} \times i = i^{2n} . i^n . i = (-1)^n . i^{n+1}$

$i^{4n+1} = (i^4)^n . i = i$

$i^{-4n+1} = \frac{1}{(i^4)^n} . i = i$

★ **Complex no.:-** Any no. of the form $(a+ib)$ where a, b, is called complex no.

If $z = a+ib$ then $\bar{z} = a-ib$ is called conjugate complex number of z.

$\bar{z} = a-ib = a+(-i)b$.

$\bar{(\bar{z})} = a+\{(-i)\}b = a+ib = z$

$z = r(\cos\theta + i\sin\theta)$

$\bar{z} = r(\cos\theta - i\sin\theta)$

★ Let $(a+ib)$ and $(a-ib)$ be any two conjugate complex nos.

(i) $(a+ib)+(a-ib) = 2a$, purely real.

(ii) $(a+ib)-(a-ib) = 2ib$,, ,, imaginary.

(iii) $(a+ib)(a-ib) = a^2 - i^2b^2 = a^2+b^2$

(iv) $\frac{a+ib}{a-ib} = \frac{(a+ib)(a+ib)}{(a-ib)(a+ib)}$

$= \frac{a^2+iab+iab+i^2b^2}{a^2-i^2b^2}$

$= \frac{a^2-b^2+i2ab}{a^2+b^2}$

$= \frac{a^2-b^2}{a^2+b^2} + i\frac{2ab}{a^2+b^2}$

★ Let $(a+ib)$ and $(c+id)$ be any two complex number

(i) $(a+ib)+(c+id) = (a+c)+i(b+d)$

(ii) $(a+ib)-(c+id) = (a-c)+i(b-d)$

(iii) $(a+ib)(c+id) = ac+iad+ibc+i^2bd$

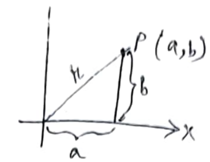
$= (ac-bd)+i(ad+bc)$

(iv) $\frac{a+ib}{c+id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)}$

$= \frac{ac-iad+i\ \ +bd}{c^2+\ }$

$= \frac{(ac+bd)-i(\ \ -bc)}{c^2+d^2}$

$\frac{ac+bd}{c^2+d^2} - i\frac{ad-bc}{c^2+d^2}$



$a+ib \Rightarrow a =$ real

$b =$ imaginary

The z plane where the complex numbers are plotted is called the Argand plane

Polar form $(r\cos\theta, r\sin\theta)$.

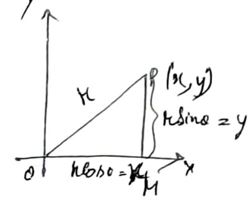
Polar form of a complex number.

Let $z = x+iy \longrightarrow (1)$ be any complex no.

Let P (x,y) be corresponding point

Let $x = r\cos\theta \longrightarrow (2)$

$y = r\sin\theta \longrightarrow (3)$



Square and added eq^n (2) and (3)

$x^2 = r^2\cos^2\theta$

$y^2 = r^2\sin^2\theta$

$x^2+y^2 = r^2(\cos^2\theta+\sin^2\theta)$

$x^2+y^2 = r^2$

$r = \sqrt{x^2+y^2} \longrightarrow (4)$

The number, r complex no. By dividing eq^n (3) and (2) we have

$\frac{y}{x} = \tan\theta$

$\theta = \tan^{-1}(y/x) \longrightarrow (5)$

The angle θ in (5) is called the Argument (or amplitude) [in (4)] is called modulation of the the mod $z = r = |z| \longrightarrow (4)$

Arg $z = \theta \longrightarrow (5)$

(r,θ) is the polar condition (or amplitude) of the ~~compa~~ complex no z.

Q Find the modulus of $2+3i$, $2-i$, $4i$

(i) $z = 2+3i$

$|z| = \sqrt{2^2+3^2}$

$= \sqrt{4+9} = \sqrt{13}$

$z = 2-i$

$|z| = \sqrt{2^2+(-1)^2}$

$= \sqrt{5}$

$z = 4i$

$|z| = 0+4i$

$= \sqrt{0^2+4^2}$

$= \sqrt{16} = 4$

(ii) $\frac{2+3i}{1-i}$

$= \frac{(2+3i)(1+i)}{(1-i)(1+i)}$

$= \frac{2+2i+3i-3}{1+1}$

$= \frac{5i-1}{2}$

$|z| = \sqrt{(-1/2)^2+(5/2)^2}$

$= \sqrt{\frac{1+25}{4}} = \sqrt{26}/2$

Q $\sqrt{x+iy} = a+ib$

Then $\sqrt{x-iy} = a-ib$

Soln:- Given

$\sqrt{x+iy} = a+ib$

Squaring both sides, we have

$x+iy = a^2+(ib)^2+2aib$

$\Rightarrow x+iy = (a^2-b^2)+i\,2ab$

Equating the real and imaginary separately, we have

$a^2-b^2 = x \rightarrow (1)$, $y = 2ab \rightarrow (2)$

$\sqrt{x-iy} = \sqrt{(a^2-b^2)-i(2ab)}$ (From (1) and (2))

$= \sqrt{a^2+(-b)^2-2aib}$

$= \sqrt{a^2+(ib)^2-2a(ib)}$

$= \sqrt{(a-ib)^2}$

$= a-ib$

Proved

(2) Find the square root of $-7+24i$

$\sqrt{-7+24i} = x+iy$

Squaring both sides, we have

$(\sqrt{-7+24i})^2 = (x+iy)^2$

$\Rightarrow -7+24i = x^2-y^2+i\,2xy$

Equating the real & the imaginary part separately we have

$x^2-y^2 = -7 \longrightarrow (2)$

$2xy = 24 \longrightarrow (3)$

x, y must have same sign.

$(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$= (-7)^2 + (2xy)^2$

$= 49+(24)^2 = 49+576$

$= 625$

$= 25^2$

$x^2+y^2 = 25 \longrightarrow (4)$

$(2)+(4) \Rightarrow$

$2x^2 = 18$

$\Rightarrow x = \pm 3$

$2xy = 24$

$\Rightarrow \cancel{18}\,6y = \cancel{24}\,4$

$\Rightarrow y = \pm 4$

$\sqrt{-7+24i} = \pm 3 + i(\pm 4)$

$= \pm 3 \pm 4i$

$= \pm(3+4i)$

(5) Find the square root of $3+4i$

Let $\sqrt{3+4i} = x+iy \longrightarrow (1)$

$\Rightarrow 3+4i = (x^2-y^2)+i\,2xy$

$x^2-y^2 = 3 \rightarrow (2)$, $2xy = 4 \rightarrow (3)$

$(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$= 3^2+4^2$

$= 25$ $\therefore x^2+y^2 = 5 \rightarrow (4)$

$(2)+(4) \Rightarrow$

$2x^2 = 8$

$\Rightarrow x^2 = 4$

$\therefore x = \pm 2$

$(3) \Rightarrow$

$2(\pm 2)y = 4$

$\Rightarrow y = \pm 1$

$\therefore \sqrt{3+4i} = \pm 2 \pm i1$

$= \pm(2+i)$

Cube roots of unity

Let x be a cube root of unity.

$\therefore x = \sqrt[3]{1}$

$x^3 = 1$

$x^3-1 = 0$

$\Rightarrow (x-1)(x^2+x+1) = 0$

Either $x-1 = 0$

$x = 1$

$x^2+x+1 = 0$

$\therefore x = \dfrac{-1 \pm \sqrt{1^2-4.1.1}}{2.1}$

$= \dfrac{-1 \pm \sqrt{-3}}{2}$

$4x^2y^2 = (2xy)^2$

The other roots are imaginary.

Let $\omega = \dfrac{-1+\sqrt{-3}}{2}$ $\omega^2 = \dfrac{-1-\sqrt{-3}}{2}$

Squaring both sides we have

$\omega^2 = \dfrac{1}{2^2}\{1-3+2(1).\sqrt{-3}\}$

$= \dfrac{1}{4}\{-2-2\sqrt{3}\}$

$= \dfrac{1}{2}\{-1-\sqrt{-3}\}$

$\omega^2 = \dfrac{1}{2}(-1-\sqrt{3}) \rightarrow (1)$

~~$(\omega^2)^2 = \frac{1}{4}(-1-\sqrt{3})^2$~~

$(\omega^2)^2 = \frac{1}{4}(-1-\sqrt{3})^2$

$\Rightarrow \omega^4 = \frac{1}{4}(1-3+2\sqrt{3})$

$= \frac{1}{4}(-2+2\sqrt{3})$

$= \frac{1}{2}(-1+\sqrt{3})$

If ω is a root of $x^2-1 = 0 \Rightarrow \omega^3 = 1$

~~$(1) \Rightarrow \omega =$~~

~~$(1) \Rightarrow$ ...~~ $-\dfrac{1}{2}$

The cube roots of unity are

$1, \omega, \omega^2$

Properties of ω :-

(1) Sum of the cube roots of unity is equal to zero
i.e. $1+\omega+\omega^2=0$

(2) $x^3-1=(x-1)(x-\omega)(x-\omega^2)$

(3) The two imaginary cube roots of unity are one is the reciprocal of the other

We have, $\omega^3=1$

$\Rightarrow \omega.\omega^2=1$

$\Rightarrow \omega^2=\frac{1}{\omega}$

or $\omega=\frac{1}{\omega^2}$

(4) The sum of the two imaginary cube roots of unity is equal to -1.

$\omega+\omega^2=-1$

Power of ω

$\omega^3=1$

(i) $\omega^4=\omega$

(ii) $\omega^5=\omega^2$

(iii) $\omega^6=(\omega^3)^2=1$

(iv) $\omega^7=\omega$

(v) $\omega^8=\omega^2$

(vi) $\omega^9=1$

(vii) $\omega^{10}=\omega$

(viii) $\omega^{3n+1}=(\omega^3)^n\cdot\omega=\omega=\omega$.

(ix) $\omega^{-3n+1}=\frac{1}{\omega^{3n}}\cdot\omega=\omega$.

Q(1) P.T. $x^3-1=(x-1)(x-\omega)(x-\omega^2)$

L.H.S $=x^3-1$

$=(x-1)(x^2+x+1)$

$=(x-1)\{x^2-(-1)x+\omega^3\}$

$=(x-1)\{x^2-(\omega+\omega^2)x+\omega^3\}$

$=(x-1)\{x^2-\omega x-\omega^2x+\omega^3\}$

$=(x-1)\{x(x-\omega)-\omega^2(x-\omega)\}$

$=(x-1)(x-\omega)(x-\omega^2)$

$=$ R.H.S

~~(2) $a^3-b^3=(a-b)^3$~~

ARITHMETIC PROGRESSION

(1) 1, 2, 3, 4, ...

(2) 2, 5, 8, ...

(3) 1, 3, 5, 7, 9 . . . . .

(4) $-11, -9, -7,$ . . . . .

(5) $a, a+d, a+2d$.

Given first term $\Rightarrow a=a+(1-1)d$

2nd " $\Rightarrow a+d=a+(2-1)d$.

3rd " $\Rightarrow a+2d=a+(3-1)d$

\- - - - - - - -

$\boxed{n\text{th} \;\; " \;\; \Rightarrow a+(n-1)d.}$

Defn :- The arithmetic progression is the succession of quantities called terms, each of which after the 1st is obtained from the preceding one by adding to it a fixed number.

Common difference (c.d).

Q(1) Find the 36th term of 3, 8, 13, . . . , find also nth term

Soln : $d=8-3=5$, $a=3$

$\therefore$ 36th term $=3+(36-1).5$

$=3+35\times5$

$=3+175=178$

nth term $=3+(n-1)5$

Q.② If x, y, z be the pth, qth and the rth terms respectively of an A.P. then

$$x(q-r)+y(r-p)+z(p-q)=0$$

Soln.-

Let the first term $= a$

common difference $= d$

By the question.

pth term $= a+(p-1)d = x \rightarrow$ ①

the qth " $= a+(q-1)d = y \longrightarrow$ ②

the rth " $= a+(r-1)d = z \longrightarrow$ ③

$x-y=(p-q)d \longrightarrow$ ④

$y-z=(q-r)d \longrightarrow$ ⑤

Deviding ④ and ⑤, we have

$$\frac{x-y}{y-z} = \frac{(p-q)d}{(q-r)d}$$

$\Rightarrow (x-y)(q-r) = (y-z)(p-q)$

$\Rightarrow x(q-r) - y(q-r) = y(p-q) - z(p-q)$

$\Rightarrow x(q-r) + y(r-p) + z(p-q) = 0$

---

③ If $2k+3$, $3k+1$ and $5k+3$ be in AP. find k.

c.d $= (3k+1)-(2k+3) = (5k+3)-(3k+1)$

$\Rightarrow k-2 = 2k+2$

$\Rightarrow k=-4$

A.M

If in an A.P. of three terms, one is called the arithmatic mean of the other two. i.e, if a, l, b are in A.P. the middle term l is called the arithmatic mean of a and b. In an A.P. the terms betn any two non consecutive terms are called the arithmetic means between these two terms.

i.e. If $a, m_1, m_2, m_3, \ldots m_n, l$ are in A.P. the terms $m_1, m_2, m_3, \ldots m_n$ are called the arithmatic means, betn a and l.

Total terms $= (n+2)$

Q. If a, b, c are in A.P. find the a.m?

$b-a = c-b$

$\Rightarrow 2b = a+c$

$\Rightarrow b = \frac{a+c}{2}$

③ In-sert n arithmatic means betn a and l,

~~Let x be the~~ a.m

$\cancel{x-a = l}$

When the n arithmatic means are in-serted the complete series will be of $(n+2)$ terms

If the first terms $= a$ (given)

The no of terms $= n+2$

The c.d $= d$

The last term $= l$.

By the question

The last term $=$ the $(n+2)$ th term.

$l = a+(n+2-1)d$.

$\Rightarrow l = a+(n+1)d$.

$\Rightarrow (n+1)d = l-a$.

$\Rightarrow d = \frac{l-a}{n+1}$

1st arithmetic mean = a+d = $a + \frac{(l-a)}{n+1}$

2nd " " = a+2d = $a + \frac{2(l-a)}{n+1}$

nth " " = a+nd = $a + \frac{n(l-a)}{n+1}$

Q. Insert 5 arithmetic mean betn 2 and 23.

Soln:

When the 5 arithmetic means ~~betn 2 & 23~~ are inserted betn 2 and 23, the complete series will be of 7 terms.

Let the first term, a = 2 (given)

The com diff = d

Total no. of terms = 7

Last term, l = 23 (given)

∴ By the question

The last term = the 7th term.

The reqd arithmetic means are

a+d, a+2d, a+3d, a+4d, a+5d, a+6d, ~~a+7d~~.

$a+(7-1)d = 23$

$\Rightarrow 2+6d = 23$

$\Rightarrow 6d = 21$

$\Rightarrow d = \frac{\cancel{21}\,7}{\cancel{6}\,2}$

$2+\frac{7}{2}$, $2+7$, $2+\frac{21}{2}$, $2+14$, $2+\frac{35}{2}$

$\Rightarrow \frac{11}{2}, 9, \frac{25}{2}, 16, \frac{39}{2}$

SUM OF THE 1ST n TERMS (of an A.P.)

Let The 1st term = a

The common diff, d

The no of terms = n

The last term = l

The sum = S

The series in their natural and reverse order

$S = a+(a+d)+(a+2d)+ \dots +(l-2d)+(l-d)+l$

$S = l+(l-d)+(l-2d)+(l-3d)+ \dots +(a+2d)+(a+d)+a$

$2S = (a+l)+(a+l)+ \dots +(a+l)$

$\Rightarrow 2S = n(a+l)$

$\Rightarrow \boxed{S = n(a+l)/2}$

l = nth term

$= a+(n-1)d$

$S = \frac{n}{2}\{a+a+(n-1)d\}$

$\boxed{S = \frac{n}{2}\{2a+(n-1)d\}}$

Q. Find Sum

(1) $1+2+3+ \dots +n$

(2) $1+3+5+ \dots +n$.

(3) $2+4+6+ \dots +n$.

Ans (1) $d = 2-1 = 1$

$a = 1$, $l = n$

~~$S = n(1+1)$~~

$S = \frac{n}{2}\{1+1+(n-1)1\}$

$= \frac{n(n+1)}{2}$

(2) $a = 1, \quad d = 2, \quad l = n.$

$$S = \frac{n}{2}\{2\times 1 + (n-1)2\}$$

$$= \frac{n}{2}\{2 + 2n - 2\}$$

$$= n^2$$

(3) $a = 2, \quad d = 4 - 2 = 2, \quad l = n$

$$S = \frac{n}{2}\{2\times 2 + (n-1)\cdot 2\}$$

$$= \frac{n}{2}\{4 + 2n - 2\} = \frac{n}{2}\times 2\{n+1\}$$

$$= \cancel{\frac{n(n+1)}{2}} = n(n+1)$$

Q. P.T.

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$a = 1^2, \quad d = 2^2 - 1^2 = 3, \quad l = n^2$

$$\therefore S = \frac{n^2}{2}\{2.1 + (n^2-1)3\}$$

$$= \frac{n^2}{2}\{2 + 3n^2 - 3\}$$

$$= \frac{n^2(3n^2-1)}{2}$$

Put

$k^3 - (k-1)^3 = 3k^2 - 3k + 1$

Put $k = 1$, $1^3 - 0 = 3.1^2 - 3.1 + 1$

$k = 2$, $2^3 - 1^3 = 3.2^2 - 3.2 + 1$

$k = 3$, $3^3 - 2^3 = 3.3^2 - 3.3 + 1$

$\dots$

$k = n-1$, $(n-1)^3 - (n-2)^3 = 3(n-1)^2 - 3(n-1) + 1$

$k = n$, $n^3 - (n-1)^3 = 3n^2 - 3n + 1$

$$n^3 = 3(1^2 + 2^2 + 3^2 + \dots + n^2) - 3(1+2+3+\dots+n) + (1+1+\dots n)$$

$$\Rightarrow n^3 = 3S - 3.\frac{n(n+1)}{2} + n$$

$$\Rightarrow 3S = n^3 + \frac{3(n+1)}{2} - n$$

$$= \frac{n}{2}[2n^2 + 3 + \dots 3n - 2]$$

$$= \frac{n}{2}\{2n^2 + 2n + n + 1\}$$

$$= \frac{n}{2}\{2n(n+1) + 1(n+1)\}$$

$$= \frac{n}{2}(n+1)(2n+1)$$

$$\Rightarrow S = \frac{n}{6}(n+1)(2n+1)$$

Q. P.T.

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$

Put

$(k+1)^2 - (k-1)^2 = 4k$

Multiply both sides by k^2, we have

$k^2(k+1)^2 - k^2(k-1)^2 = 4k^3$

Put $k = 1$, $1^2(2)^2 - 1^2.0 = 4.1^3$

$k = 2$ $\quad 2^2 3^2 - 2^2.1^2 = 4.2^3$

$k = 3$ $\quad 3^2.4^2 - 3^2.2^2 = 4.3^3$

$\dots$

$k = n-1$, $(n-1)^2 n^2 - (n-1)^2(n-2)^2 = 4(n-1)^3$

$k = n$, $n^2(n+1)^2 - n^2(n-1)^2 = 4n^3$

Put S $= n^2(n+1)^2 = 4(1^3 + 2^3 + \dots + n^3)$

$$\Rightarrow 4S = n^2(n+1)^2$$

$$\Rightarrow S = \left\{\frac{n(n+1)}{2}\right\}^2$$

GEOMETRIC PROGRESSION

1. $1, 2, 4, 8$
2. $1, 3, 9, 27$.
3. $2^2, 2^2, 2^2$
4. $a, ar, ar^2, ar^3, \ldots$
5. $2, -1, \frac{1}{2}, -\frac{1}{4}, \ldots$

(4) a, ar, ar^2, ar^3

1st term $= a = ar^{1-1}$ Common ratio $= \frac{ar}{a}$

2nd " $= ar = ar^{2-1}$ $= r$

3rd " $= ar^2 = ar^{3-1}$

$\boxed{n\text{th} \;\text{"} = ar^{n-1}}$ ★

Def^n: A geometric progression (G.P. series), is a succession of quantities called terms each of which after the 1st is formed by multiplying the preceding one by a fixed number.

Q.1. Find the 9th term of

$2, -1, \frac{1}{2}$

C.R. $= \frac{-1}{2}$. $a = 2$

$\therefore$ 9th term $= 2.\left(-\frac{1}{2}\right)^{9-1}$

$= 2\frac{1}{(-2)^8} = 2 \times \frac{1}{2^4 . 2^4}$

$= \cancel{2} \times \frac{1}{8 \times \cancel{16} \times 16}$

$= \frac{1}{128}$

Q. The 6th and the 13th term of a G.P. are 64 and 8192 respectively. Find the 10th term.

A/Q, $64 = a.r^5$ → (i)

$8192 = a.r^{12}$ → (ii)

(i)/(ii) $\Rightarrow \frac{64}{8192} = \frac{ar^5}{ar^{12}}$

$\Rightarrow 128$

$\Rightarrow 128 = r^7$

$\Rightarrow r = 2$

$\therefore$ (i) $\Rightarrow 64 = a.2^5$

$\Rightarrow a = 2$

$\therefore$ 10th term $= ar^{10-1}$

$= 2.2^9$

$= 2^{10}$

Q. The sum and the product of three numbers in G.P. are respectively 52 and 1728, find the numbers.

Let the numbers are ~~a, ar, ar^2~~ $\frac{a}{r}, ar, a$

A/Q $\therefore$ ~~$a + ar + ar^2 = 52$~~

~~$a.ar.ar^2 \Rightarrow$~~

$\frac{a}{r} + ar + a = 52$

$\frac{a}{\cancel{r}} \times a\cancel{r}.a = 1728$

$\Rightarrow a^3 = 1728 = 12^3$

$\Rightarrow a = 12$.

$\therefore \frac{12}{r} + 12r + 12 = 52$

$\Rightarrow \frac{12 + 12r^2}{r} = 40$ $\Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$\therefore r = 3,\ r = 1/3$

Numbers are $\frac{12}{3},\ 12 \cdot 3,\ 12$

$= 4, 12, 36$

and $\frac{12}{1/3},\ 12 \cdot \frac{1}{3},\ 12$

$= ~~36, 4, 12~~$

$= 36, 12, 4$

Geometric mean :-

In a G.P. of three terms the middle one is called the geometric mean of the two

i.e. if a, b, c are in G.P. the middle term b is called the G.M. of a and c.

In a G.P. the terms betn any two non consecutive terms are called geometric means betn these two terms.

i.e. if $a, g_1, g_2, g_3, \ldots g_n, l$ are in G.P. then the terms $g_1, g_2, g_3 \ldots g_n$ are called geometric mean betn a and l.

Q 4. Insert n geometric means betn a and l.

Soln:- ~~Let the~~

When the n geometric means are inserted, the complete series will be of (n+2) terms.

Let the 1st term = a (given)

The common ratio = r.

The no. of terms = (n+2)

The last term = l (given)

By the question

The last term = l

$\therefore$ The (n+2)th term = l

$\therefore ar^{n+2-1} = l$

$\Rightarrow ar^{n+1} = l.$

$\Rightarrow r^{n+1} = l/a \quad \therefore \boxed{r = (l/a)^{1/(n+1)}}$

The required n geometric means are

$a,\ a\left\{(l/a)^{\frac{1}{n+1}}\right\}^{2-1},\ a\left\{\left(\frac{l}{a}\right)^{\frac{1}{n+1}}\right\}^2,\ a\left\{(l/a)^{\frac{1}{n+1}}\right\}^3, \ldots\ldots, a\left\{(l/a)^{\frac{1}{n+1}}\right\}^{n-1},\ a\left\{(l/a)^{\frac{1}{n+1}}\right\}^n,\ l.$

(5) Insert 5 geometric mean betn 64 and $\frac{729}{64}$.

$a = 64,\ l = \frac{729}{64},\ n = 5$

$\therefore r = \left(\frac{l}{a}\right)^{\frac{1}{5+1}}$

$= \left\{\frac{729/64}{64}\right\}^{\frac{1}{5+1}}$

$= \left(\frac{729}{64 \times 64}\right)^{\frac{1}{6}}$

$r = \frac{3}{4}$

Sum of a G.P. Series (upto n terms)

Let

a = 1st term

r = the common ratio

n = the no. of terms

S = the sum.

The series will be

$S = a + ar + ar^2 + \ldots\ldots + ar^{n-1}$

$rS = ar + ar^2 + \ldots\ldots + ar^{n-1} + ar^n$

$S - rS = a - ar^n$

$\Rightarrow S(1-r) = a(1-r^n)$

$\Rightarrow S = a\frac{1-r^n}{1-r},\ r \neq 1$

Case ① If $r>1$

$\therefore S = a\dfrac{r^n-1}{r-1}$

Case ② If $r=1$

$S = a + ar + ar^2 + \dots + ar^{n-1}$

$= a + a + \dots + a$

$= na$

Case ③ If $n \to \infty$, $r<1$

$$S = \frac{a(1-r^n)}{1-r}$$

$r = \frac{1}{2}$, $r^{10} = \frac{1}{2^{10}}$, $r^{100} = \frac{1}{2^{100}}$

$r^n = \frac{1}{2^n} = \frac{1}{\infty} = 0$

$\therefore S = \dfrac{a(1-0)}{1-r} = \dfrac{a}{1-r}$

Q. Sum the G.P. series.

(i) $1+3+9+\dots$ to 9 term.

(ii) $6+4+\frac{8}{3}+\frac{16}{9}\dots$ to ∞

(i) $S = \dfrac{a(1-r^n)}{1-r}$

$a=1$, $r=\frac{3}{1}=3$, $n=9$

$\therefore S = \dfrac{1(1-3^9)}{1-3}$

$= \dfrac{1-19683}{-2}$

$= \dfrac{19682}{2}$

$= \cancel{9791}$ 9841

$$\begin{array}{r} 27 \\ \times 27 \\ \hline 189 \\ 54 \\ \hline 729 \\ \times 27 \\ \hline 5103 \\ 1458 \\ \hline 19683 \end{array}$$

(ii) $a=6$, $r=\frac{4}{6}=\frac{2}{3}$, $n=\infty$

$S = \dfrac{a}{1-r}$

$= \dfrac{6}{1-\frac{2}{3}}$

$= \dfrac{18}{1}$

$= 18$

② Sum of the G.P. series

(i) $\frac{1}{16}+\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+\dots+128$.

(ii) $\sqrt{2}+\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}+\dots$ to ∞

1st term $a = \frac{1}{16}$, $r = \dfrac{1/8}{1/16} = \frac{16}{8} = 2 > 1$

$128 = ar^{n-1}$

$128 = \frac{1}{16}\times 2^{n-1}$

$\Rightarrow 16\times 128 = 2^{n-1}$

$\Rightarrow 2^4\times 2^7 = 2^{n-1}$

$\Rightarrow n-1 = 11$

$\Rightarrow n = 12$.

$\therefore S = a\times\dfrac{r^n-1}{r-1}$

$= \dfrac{1}{16}\times\dfrac{2^{12}-1}{2-1}$

$= \dfrac{2^{12}-1}{16}$

(ii) $a = \sqrt{2}$ $\quad r = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$ $\quad n \to \infty$

$$\therefore S = \frac{a}{1-r}$$

$$= \frac{\sqrt{2}}{1-\frac{1}{2}}$$

$$= 2\sqrt{2}$$

Σ notation is used to sum of similar terms.

$$\sum n = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = 1^3+2^3+3^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum (ax+bx^2) = \sum ax + \sum bx^2$$

$$= a\sum x + b\sum x^2 = a$$

$$\sum x+x^2+c = \sum x + \sum x^2 + \sum c +$$

$$= \sum x + \sum x^2 + c\sum 1$$

Q. Sum of n terms.

$1.2+2.3+3.4+\ldots$ to n term

$2(1+3+3.2$

$= 2+6+12+\ldots$ to n terms.

1st term, $a = 2$

Common ratio $r = \frac{6}{2} = 3$

The first factor of the given series is $1, 2, 3, \ldots$ to n terms form an A.P. whose 1st term $a=1$. The c.d $=1$, the nth term $= a+(n-1)d$
$= a+(n-1).1 = n$.

Ily the 2nd second factor of the above series is $2, 3, 4$ to n terms where 1st term $a=2, d=1$, the nth term $= n+1$

The nth term of the given series will be $n(n+1)$.

Let the sum be S.

$$S = \sum n(n+1)$$

$$= \sum n^2 + \sum n.$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1)+3n(n+1)}{6}$$

$$= \frac{n(n+1)\{(2n+1)+3\}}{6}$$

$$= \frac{n(n+1)(2n+4)}{6}$$

$$= \frac{n(n+1)(n+2)}{3}$$

Q. Sum of n terms.

$1.2.3+2.3.4+3.4.5+\ldots$ to n terms

The first factor of this series $1, 2, 3, \ldots n$.

$\therefore$ nth term $= a+(n-1)d$
$= 1+(n-1).1 = n$.

The second factor of this series $2, 3, 4, \ldots$ to n ter

$\therefore$ nth term $= a+\{(n+1)-1\}d$.
$= 2+(n+1-1)1$
$= n+1$

LOGARITHM

Let a, x, N be three numbers such that $a^x = N$ $(a>0, a\neq 1)$ then the index x is called the logarithm of the number N to the base a.

$$\log_a N = x$$

$2^4 = 16$, $4^2 = 16$.

$\log_2 16 = 4$ $\quad \log_4 16 = 2$.

$3^4 = 81$ $\quad 9^2 = 81$

$\Rightarrow \log_3 81 = 4$ $\quad \Rightarrow \log_9 81 = 2$.

$9^{-1/2} = \frac{1}{3}$

$\Rightarrow \log_9 \frac{1}{3} = -\frac{1}{2}$

$\log (\frac{1}{2}) = \log 2^{-1}$

$= -1 \log 2$

Q. P.T. (1) $\log_a 1 = 0$ (2) $\log_a a = 1$ (3) $a^{\log_a N} = N$.

Soln:-

$a^0 = 1$

By defn, we have

$\log_a 1 = 0$.

(2)

$a^1 = a$

$\therefore \log_a a = 1$

(3) Let a, x, N be any three numbers.

Such that $a^x = N$ $\begin{bmatrix} a>0 \\ a\neq 1 \end{bmatrix} \longrightarrow (1)$

$\log_a N = x$. $\longrightarrow (2)$.

Putting this value of x from (2) in (1)

$a^{\log_a N} = N$

Laws of Logarithm :-

(1) $\log_a (m \times n) = \log_a m + \log_a n$.

(2) $\log_a (m/n) = \log_a m - \log_a n$

(3) $\log_a (m^n) = n \log_a m$.

(1) $\log_a m = x \rightarrow (1)$ and $\log_a n = y \longrightarrow (2)$

$\Rightarrow a^x = m, \rightarrow (3)$ $\quad a^y = n \rightarrow (4)$

~~$x \cdot y = \log_a m \times \log$~~

(3) $\times$ (4) $\Rightarrow a^{x+y} = mn$

$\Rightarrow \log_a mn = x+y$

$= \log_a x + \log_a y$

(2) Let $\log_a m = x$, $\quad \log_a n = y$

$\therefore a^x = m$, $\quad a^y = n$

$\therefore \frac{a^x}{a^y} = \frac{m}{n}$

$\Rightarrow a^{x-y} = m/n$

$\Rightarrow \log_a (m/n) = x-y$

$\boxed{\log_a (m/n) = \log_a m - \log_a n}$

(3) Let $\log_a (m^n) = x \rightarrow (1)$ $\quad \log_a m = y \longrightarrow (2)$

$\Rightarrow a^x = m^n \rightarrow (3)$ $\quad \Rightarrow a^y = m \longrightarrow (4)$

~~$x+y = m^{n+1}$~~ . $a^x = m^n$

$= (a^y)^n$

$a^x = a^{yn}$

$\Rightarrow x = yn$.

$\Rightarrow \log_a (m^n) = n \cdot \log_a m$

1. Find the logarithm of 400 to the base $(2\sqrt{5})$.

Let $\log_{2\sqrt{5}} 400 = x$

$\therefore (2\sqrt{5})^x = 400$

$\Rightarrow \sqrt{20}^{\,x} = (\sqrt{20})^4$

$= \sqrt{20}.\sqrt{20}.\sqrt{20}.\sqrt{20}$

$\therefore x = 4$

2. Find the logarithm of $\overline{144}$ to the base $2\sqrt{3}$

Let $\log_{2\sqrt{3}} 144 = x$

$\Rightarrow \sqrt{12}^{\,x} = (\sqrt{12})^4$

$x = 4$

3. P.T

$7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80} = \log 2$

L.H.S $= 7\log\frac{16}{15} + 5\log\frac{25}{24} + 3\log\frac{81}{80}$

$= 7\log 2^4 - 7\log(3\times5) + 5\log 5^2 - 5\log(3\times8)$

$+ 3\log 3^4 - 3\log(8\times$

$= 28\log 2 - 7\log 3 - 7\log 5 + 10\log 5 - 5\log 3 - 5\log 2^3$

$+ 12\log 3 - 9\log 2 - 3\log 2 - 3\log 5$

$= 28\log 2 - 15\log 2 - 9\log 2 - 3\log$

$- 7\log 3 + 5\log 3 + 12\log 3 \quad 7\log 5 + 10\log 5 + 3\log 5$

$= \log 2$ ~~$-\log 3 \ldots$~~

$= \log 2$

Q. P.T. $\log_a b \times \log_b a = 1$

Let $\log_a b = x \rightarrow (1)$ $\qquad \log_b a = y \rightarrow (2)$

By def^n $\Rightarrow a^x = b \longrightarrow (3)$ $\qquad \Rightarrow b^y = a \rightarrow (4)$

From (3) and (4)

$a^x = b$

$\Rightarrow (b^y)^x = b$

$\Rightarrow b^{yx} = 1$

$\Rightarrow xy = 1$

$\Rightarrow \log_a b \times \log_b a = 1$

Q. Find the sum of the series.

1. $1 + 5 + 9 + \ldots + 85$
2. $2 + 5 + 8 + \ldots$ to 31 terms

(1) First term, $a = 5$

comm. diff., $d = 5 - 1 = 4$

Last term $l = 85$

Let no. of terms, n

$\therefore a + (n-1)d = 85$

$\Rightarrow 5 + (n-1).4 = 85$

$\Rightarrow (n-1)4 = 80$

$\Rightarrow n = 20 + 1$

$= 21$

$\therefore S = \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{21}{2}\{2\times1 + 20\times4\}$

$= \frac{21}{2}\{2 + 80\} = 21\times41 = 861$

$\begin{array}{r} 21 \\ 41 \\ \hline 21 \\ 84 \\ \hline 861 \end{array}$

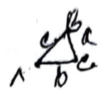
MENSURATION

Geometrical branch of Mathematics.

Geometrical branch of Mathematics, Volumes, Surface area, of solid figure, Area of plan fig. bounded by straight lines, strains,

① Plane figure → bounded by straight lines,
triangle, circle

① Circle — Area $= \pi r^2$, $P = 2\pi r$

② Triangle 

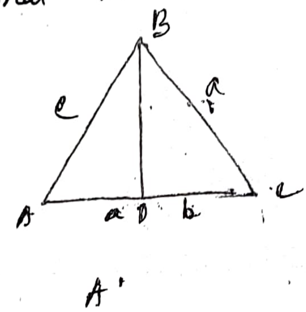
Area $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times b \times h$

$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \dfrac{a+b+c}{2}$

③ Equilateral Δle



$AD = \dfrac{a}{2} = CD.$

From Δ BCD

$BC^2 = BD^2 + CD^2$

$\Rightarrow a^2 = (a/2)^2 + h^2$

$\Rightarrow h^2 = \dfrac{4a^2 - a^2}{4}$

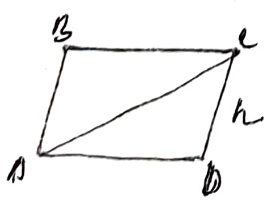
$\therefore h = \dfrac{\sqrt{3}a}{2}$

Area $= \frac{1}{2} \times a \times h$

$= \frac{1}{2} \times a \times \dfrac{\sqrt{3}.a}{2}$

$= \dfrac{\sqrt{3}}{4} a^2$

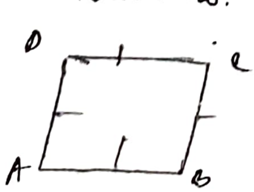
④ Parallalogram. (opp. sides are equal and ∥)



Area $= 2 \left[\frac{1}{2} \times \text{base} \times h\right]$

$=$ base $\times h$.

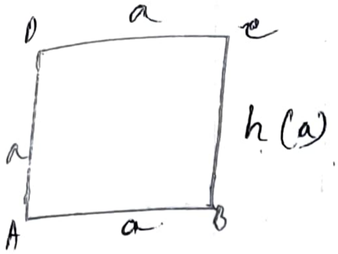
⑤ Rhombus. (opp. Angles and Sides are equal)



Area $= \frac{1}{2} \times$ base $\times h$

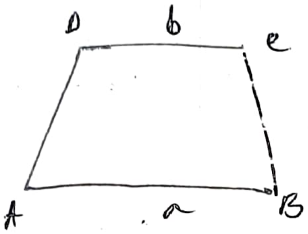
$= \frac{1}{2} \times AB \times BC$

⑥



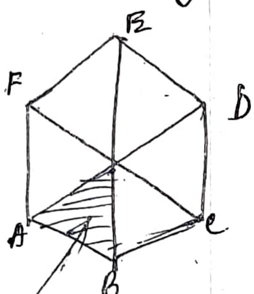
$= a \times a$

⑦ Trapezium



Area $= \frac{1}{2}(a+b)h$,

⑧ Hexagon.

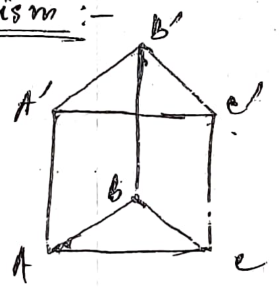


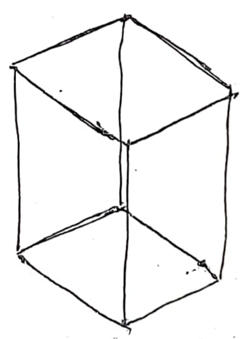
Equilateral. △

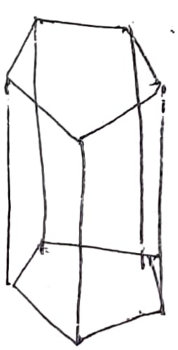
Area $= \cancel{6}^{3} \times \frac{\sqrt{3}}{\cancel{4}_{2}} \times a^2$

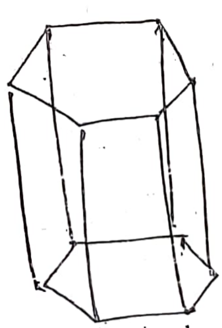
$= \frac{3\sqrt{3}}{2} a^2$

⑨ <u>Prism</u> :-









Defⁿ:- T

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Def^n :- It is a solid whose all side faces are parallalogram and whose two ends are equal, ∥ and rectilineal figure.

(1) Lateral edge (e) :- The lines of intersections of the side faces are called the lateral edge,

(2) Height (h) : The ⊥^{r} distance bet^n the two ∥ ends is called the height

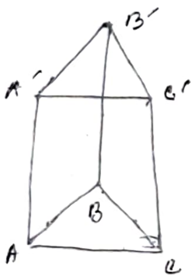
(3) The straight line joining the centres of two ends is called the axis (ABC, A'B'C')

(4) Right Prism :- If the lateral edge are ⊥ to the base then that prism is called the right prism.

(5) Volume : Area of the base × ht

= Ah.

Q.1. The base of the a right angle prism 50 cm height is a equilateral triangle on a side of 5 cms. Find its lateral side



Let ABC be the equilateral triangle base h, the ht, the lenght of the side of base ~~Sum~~ of the gives 1st prism. the length of the lenght of the side

$h = 50$ cm (given)

$a = 5$ cm

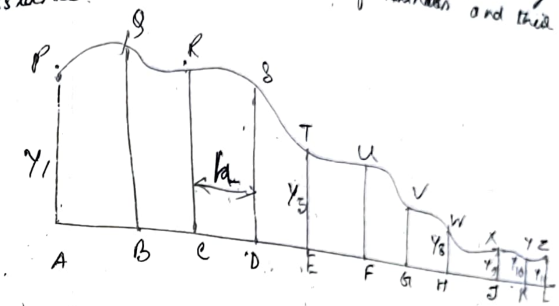
P = perimeter of the base.

$P = a + a + a = 3a = 3 \times 5 = 15$ cm.

$L.S. = P.h$ where P = per.

$= 15 \times 50$

To find the approximately the area of a figure of which one of the boundary lines is an irregular curve, having given the lengths of an odd number of ordinates and their common distance.



AL = Base.

Let the figure is devided into an even number (say, 10) of equal parts by a series of odd number of 11 ordinates.

Let h = the common distance.

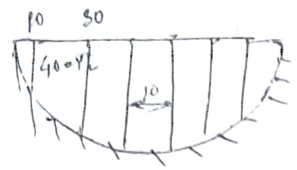
Approximate area $= \frac{h}{3}\left[Y_1 + Y_{11} + 2(Y_3 + Y_5 + Y_7 + Y_9) + 4(Y_2 + Y_4 + Y_6 + Y_8 + Y_{10})\right]$

$= \frac{\text{comm. distance}}{3}\left[(\text{1st ordinate} + \text{last ordinate}) + 2(\text{Sum of the remaining odd ordinate}) + 4(\text{" " " even ordinate})\right]$

Q. A river is 80 ft wide, its depth at a distance of x ft. from one bank is d ft and is given by the following table. below.

x =	0	10	20	30	40	50	60	70	80
d :	0	40	70	94	121	153	142	86	31

Find the sim Rules the area of the cross section of the river.



Solⁿ..

Let the common distance, $h = 10$ ft.

Also

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
0	40	70	94	121	153	146	86	31

Area (approx) $= \frac{h}{3}\left[(y_1+y_9)+2(y_3+y_5+y_7)+4(y_2+y_4+y_6+y_8)\right]$

$= \frac{10}{3}\left[(0+31)+2(40+\cancel{70}+121+146)+4(40+94+153+86)\right]$

$= \frac{10}{3}\left[31+2\times 307 + 367\times 4\right]$

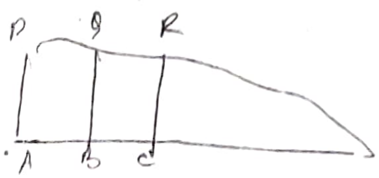
$= \frac{10}{3}\left[31+614+1468\right]$

$= \frac{10}{3}\times 2113$

$= \frac{21130}{3} = 7043.33$

(2) The ordinate of an irregular rectilinear figure measure 0, 10, 12, 16, 21, 19, 15, 11 and 7 ft. Find by simpson's rule, the area of the figure, The common distance betⁿ the ordinates being 6 ft.

y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
0	10	12	16	21	19	15	11



Area $= \frac{6}{3}\left[(0+7)+2(12+21+15)+4(10+16+19+11)\right]$

$= 2\left[7+48\times 2+4\times 56\right]$

$= 654$

Q. Find by Simpson Rule, the area of a curvilinear figure, whose ordinates measure 18, 22, 25, 24, 20, 28, 30, 34, 28, 24 and ~~of the figure~~ whose base is 146 cm ~~and whose~~

Comm distance $= \frac{146}{10} = 14.6$.

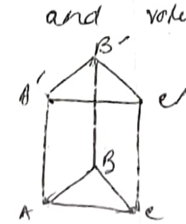
$\therefore$ Area $= \frac{14.6}{3}\left[(18+24)+2(26+20+30+28)+4(22+24+26+34)\right]$

$= 3650$

Q Applying Simpsons Rule, find the area of a field having the ordinates 0, 6, 10, 17, 25, 38, 48, 37, 28, 21, 15, 7 and 0 ms and common distance betⁿ them is 33 cms.

Q. The ordinates of an irregular rectilinear figure measure 0, 10, 12, 16, 21, 19, 15, 11 and 7 ft. Find by Simpsons Rule the area of the figure, the common distance betⁿ the ordinates being 5 ft.

Q. The perimeter of the base of a rt prism standing on an equilateral triangle is 12 cms. If the length of the prism be 60 cms, find its whole surface and volume.



Perimeter $= 12$ cms

Length of prism $= 60$ cms.

$a+a+a = 12$

$\Rightarrow 3a = 12$

$a = 4$

$a =$ length of the side of base.

MATHEMATICS

2011 (N)

(a) If $A = \{x : x \in R, x^2 - 1 = 0\}$
$B = \{x : x^2 + 3x + 2 = 0, x \in R\}$
Write $A \cup B$ in tabular form.

Ans. $A = x = \pm 1$

B: $x^2 + 3x + 2 = 0$

$\Rightarrow (x+2)(x+1)$

$\Rightarrow x = \frac{-3 \pm \sqrt{9-8}}{2}$

$= \frac{-3 \pm 1}{2}$

$= -2, -1$

$A \cup B = \{-2, -1, +1, 2\}$

(b) Find the modulus of $\frac{2+3i}{1-2i}$

Ans. $\frac{2+3i}{1-2i} = \frac{(2+3i)(1+2i)}{(1-2i)(1+2i)}$

$= \frac{2+4i+3i-6}{1+4}$

$= \frac{-4+7i}{5}$

$= 7/5 \; 4/5$

$\left|\frac{2+3i}{1-2i}\right| = \sqrt{(7/5)^2 + (-4/5)^2}$

$= \sqrt{\frac{49+16}{25}} = \sqrt{\frac{65}{25}} = \frac{\sqrt{65}}{5}$

(c) Show that there are no terms independent of x in the expansion of $(x + 1/x)^{19}$

(d) If $x^2 \propto y^2$ and $x = 1$ when $y = 1$ find x when $y = 2$

Sol^n: $x^2 \propto y^2$

$\Rightarrow x^2 = ky^2$ where k is proportionality constant

when $x = 1$, $y = 1$

$1^2 = k \cdot 1^2$

$\Rightarrow k = 1$

when $y = 2$

$x^2 = 1 \times (2)^2$

$= 4$

$\therefore x = \pm 2$ Ans

(e) Evaluate

$\log_2 \log_2 \log_3 81$

$= \log_2 \log_2 \log_3 3^4$

$= \log_2 \log_2 4 \log_3 3$

$= \log_2 \log_2 2^2$

$= \log_2 2 \quad = 1$

(f) If ${}^nC_5 = {}^nC_{12}$, find n.

(g) Find four arithmetic mean betn 3 and 23

When four arithmatic mean inserted betn 3 and 23. Then total no of term will be 6.

First term, $a = 3$

Let comm diff $= d$.

Last term, $l = 23$

$l = a+(6-1)d$

$\Rightarrow 23 = 3+5d$

$\Rightarrow 5d = 20$

$\Rightarrow d = 4$

The arithmetic means are $(a+d), (a+2d); (a+3d); (a+4d)$

$= (3+4); (3+2\times4), (3+3\times4); (3+4\times4)$

$= 7, 11, 15, 19$

(2) @ If α, β are the roots of $x^2-px+q=0$, $q\neq0$, obtained the quadratic equation whose roots are α/β and β/α

$\because$ α, β are the roots of $x^2-px+q=0$

$\therefore$ $\alpha+\beta=p$; $\alpha\beta=q$

Quadratic equation which have the roots α/β and β/α

$x^2 - \left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x + \cancel{\frac{\alpha}{\beta}}\times\cancel{\frac{\beta}{\alpha}} = 0$

$\Rightarrow x^2 - \left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)x+1=0$

$\Rightarrow x^2 - \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}x+1=0$

$\Rightarrow x^2 - \frac{p^2-2q}{q}x+1=0$

$\Rightarrow qx^2-(p^2-4q)x+q=0$

(b) If $x=1-i$ find the value of x^2-2x+2

x^2-2x+2

$= (1-i)^2 - 2(1-i)+2$

$= \cancel{1}+i^2-2\cancel{i}-\cancel{2}+\cancel{2i}+\cancel{2} = 0$

(b) @ If a, b, c be the pth, qth rth term of a G.P.

P.T. $a^{q-r} b^{r-p} c^{p-q} = 1$

Let common ratio of the G.P is r and first term is x

$x r^{p-1} = a$

$x r^{q-1} = b$

$x r^{r-1} = c$

L.H.S. $= a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$

$= (x r^{p-1})^{q-r} (x r^{q-1})^{(r-p)} (x r^{r-1})^{(p-q)}$

$= x^{\cancel{q}-\cancel{r}+\cancel{r}-\cancel{p}+\cancel{p}-\cancel{q}} r^{\cancel{pq}-\cancel{pr}-\cancel{q}+\cancel{r}+\cancel{qr}-\cancel{qp}-\cancel{r}+\cancel{p}+\cancel{rp}-\cancel{rq}-\cancel{p}+\cancel{q}}$

$= x^0 \cdot r^0 = 1.1 = 1$ = R.H.S.

$^nP_3 = 336$ find $^nC_3$

$^nP_3 = 336$, $^nC_3 = \frac{\lfloor n}{\lfloor n-3 \lfloor 3}$

$= \frac{^nP_3}{\lfloor 3}$

$= \frac{\cancel{336}\ 112\ 56}{\cancel{3}\times\cancel{2}\times1}$

$= 56$ Ans

$^nC_r = \frac{\lfloor n}{\lfloor r \lfloor n-r}$

$^nP_r =$

4 (a) Find the coeff of x^9 in the expansion of $\left(2x^2+\frac{1}{x}\right)^{20}$

(b) Evaluate $\begin{vmatrix} 23 & 13 & 4 \\ 42 & 30 & 12 \\ 67 & 52 & 16 \end{vmatrix}$

5 (a) P. T.

$$\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x).$$

(a) Show that

$$\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\dots = \log_e 2$$

$$\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$$

Putting $x=1$

$$\log(1+1) = 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\dots$$

$$\Rightarrow \log 2 = (1-1/2)+(1/3-1/4)+(1/5-1/6)$$

$$= \left(\frac{2-1}{1.2}\right)+\left(\frac{4-3}{3.4}\right)+\left(\frac{6-5}{5.6}\right)+\dots$$

$$= \frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\dots$$

6 (b) Show the $e^x+e^{-x} = 2\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\dots\right)$

$$e^x = 1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\dots$$

$$e^{-x} = 1-\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}-\frac{x^3}{\lfloor 3}+\dots$$

$$e^x+e^{-x} = 2\left(1+\frac{x^2}{2!}+\frac{x^{4}}{4!}+\dots\right)$$

(b) If $A=\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}_{2\times2}$ and $B=\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}_{2\times2}$ find $A\times B$.

Write down the 6 th term in the expansion of $(x^2-2x)^{10}$

(a) Find the value of $\cos(-1290^\circ)$

$$\cos(-1290^\circ)$$

$$= \cos(1290^\circ)$$

$$\begin{array}{r} 129 \\ 108 \\ \hline 21 \end{array}$$

$$= \cos(90^\circ\times12+210^\circ)$$

$$= \cos 210^\circ$$

$$= \cos(90^\circ\times2+30^\circ)$$

$$= -\cos 30^\circ$$

$$= -\sqrt{3}/2$$

If $\sin\theta=\frac{4}{5}$ find $\sec\theta$ and $\operatorname{cosec}\theta$

$\sin\theta = \frac{4}{5}$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\sqrt{1-\sin^2\theta}} = \frac{1}{\sqrt{1-(4/5)^2}} = \frac{5}{3}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

(b) Show that $\cos 3\alpha = 4\cos^3\alpha-3\cos\alpha$

$$\cos 3\alpha = \cos(2\alpha+\alpha)$$

$$= \cos2\alpha.\cos\alpha-\sin\alpha.\sin2\alpha$$

$$= (2\cos^2\alpha-1)\cos\alpha - 2\sin\alpha.\cos\alpha \sin\alpha$$

$= 2\cos 3\alpha - \cos\alpha - 2\cos^2\alpha \cos \sin^2\alpha$

$\Rightarrow 2\cos 3\alpha - \cos\alpha - 2\cos\alpha(1-\cos^2\alpha)$

$\Rightarrow 2\cos 3\alpha - \cos\alpha - 2\cos\alpha + 2\cos 3\alpha$

$\Rightarrow 4\cos 3\alpha - 8\cos\alpha$

(8) (d) P.T. $\tan^{-1}x = \sec^{-1}\sqrt{1+x^2}$

Let $\tan\theta = x$

$\Rightarrow \tan^{-1}x = \theta$

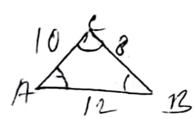
$\because 1+\tan^2\theta = \sec^2\theta$

$\Rightarrow \sec\theta = \sqrt{1+x^2}$

$\Rightarrow \theta = \sec^{-1}\sqrt{1+x^2}$

$\therefore \tan^{-1}x = \sec^{-1}\sqrt{1+x^2}$

(e) The sides of a triangle are 8cm, 10cm. and 12cm. find the greatest angle.



(f) Evaluate $\sin 15^\circ$

$\theta = 15^\circ$

$\Rightarrow 2\theta = 30^\circ$

$\Rightarrow \cos 2\theta = \cos 2\theta$

$\Rightarrow \cancel{\cos 30} \; 1 - 2\sin^2 15^\circ = \frac{\sqrt{3}}{2}$

$\Rightarrow 2\sin^2 15 = 1 - \frac{\sqrt{3}}{2}$

$\Rightarrow \sin^2 15 = \frac{2-\sqrt{3}}{2\sqrt{2}}$

$\sin 15 = \sqrt{\frac{2-\sqrt{3}}{2\sqrt{2}}}$

$= \sin(45^\circ - \cancel{\cos}\,30^\circ)$

$= \sin 45^\circ\cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

(3) (a) If $\tan\theta = \frac{a}{b}$ show that

$\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b}$

$= \frac{a \times \frac{a}{b} - b}{a \times \frac{a}{b} - b}$

$= \frac{a^2 - b^2}{a^2 + b^2}$

(b) If $A+B+C = \pi/2$ P.T.

$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

$A+B+C = \pi/2$

$\Rightarrow \tan(A+B+C) = \tan \pi/2$

$\Rightarrow \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A} = \infty$

$\therefore 1 - \tan A \tan B - \tan B \tan C - \tan C \tan A = 0$

$\therefore \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

(9) (a) P.T. $\frac{\sin\theta + \sin 2\theta}{1+\cos\theta + \cos 2\theta} = \tan\theta$

L.H.S. $= \frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta}$

$= \frac{\sin\theta + 2\sin\theta\cos\theta}{\cancel{1} + \cos\theta + 2\cos^2\theta - \cancel{1}}$

$= \frac{\sin\theta(1 + 2\cos\theta)}{\cos\theta(1 + 2\cos\theta)}$

$= \tan\theta$

$=$ R.H.S.

(B) P.T. $3\tan^{-1}x = \tan^{-1}\frac{3x-x^3}{1-3x^2}$

Let $\tan^{-1}x = \theta$

$\Rightarrow \tan\theta = x$

We have

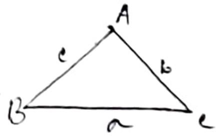
$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

$= \frac{3x - x^3}{1-3x^2}$

$\Rightarrow 3\theta = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$\Rightarrow 3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

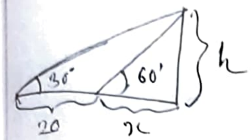
(b) If a, b, c measure three sides BC, CA, AB of a triangle ABC, prove that $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$



$L.H.S = a\sin(B-C) + b\sin(C-A) + c\sin(A-B)$

$= a\sin$

The angle of elevation of the top of a tower is 30°. On walking 20 metres nearer the elevation is found to be 60°. Find the height of the tower.



~~Since~~

~~$\cos 30° = \frac{20+x}{h}$~~

~~$\Rightarrow \frac{\sqrt{3}}{2} = \frac{20+x}{h}$~~

~~$\cos 60° = \frac{x}{h}$~~

$\tan 60° = \frac{h}{x}$

$\Rightarrow h = \sqrt{3}x \to (1)$

$\tan 30° = \frac{h}{20+x}$

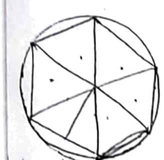
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{20+x}$

$\Rightarrow 20 + x = 3x$

$\Rightarrow x = 10$

(1) $\Rightarrow h = 10\sqrt{3}$ m

(a) Find the area of a regular hexagon inscribed in a circle of radius 6cm.



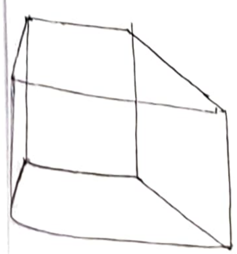
$\frac{\sqrt{3}a^2}{2}$

Area of the hexagon $= 6\times\frac{\sqrt{3}a^2}{4}$

$= 6\times\frac{\sqrt{3}}{4}\times 6^2 9$

$= 54\times\sqrt{3}$ cm²

(b) The base of a prism of height 24cm is a trapezoid whose parallel sides are 26cm and 84cm. If the distance between the ∥ sides be 18cm, find the volume of the prism.



Volume = Area of the base × height

$= \frac{1}{\cancel{2}}(26+34) \times 18 \times \cancel{24}^{12}$

$= 12 \times 18 \times 60$

$= 720 \times 18$

$= 12960\ cm^3$ Ans

(a) How many square metres of canvass is used in a conical Tent whose height is 35 metres and radius of the base is 84 metres?

Total surface area of the tent = Area of base. + Area of curved surface

$= \pi r^2 + \pi r l \longrightarrow (i)$

$\because\ l^2 = h^2 + r^2$

$\Rightarrow l^2 = 35^2 + (84)^2$

$= 8281$

$\therefore l = \sqrt{8281} = 91$ metre

$(i) \Rightarrow A = \pi (84)^2 + \pi \times 84 \times 91$

$= \frac{22}{\cancel{7}} \times \cancel{84}^{12} \times 84 + \frac{22}{\cancel{7}} \times \cancel{84}^{12} \times 91$

$= 22 \times 12 \times 84 + 22 \times 12 \times 91$

$= 22176 + 24024$

$= 46200\ m^2$

$\therefore$ $46200\ m^2$ canvass ~~is~~ required.

(12) (a) Find the amount of water a bucket in the form of a frustum of a cone can ~~can~~ hold if the raddi of the top and bottom bases are 22 cm and 14 cm and the height of the bucket is 12 cm.

Radius of top, r_1 = 22 cm.

$\therefore$ Area of the top $A_1 = \pi . 22^2$ cm^2

Radius of bottom, R = 14 cm

$\therefore$ Area of the bottom $A_2 = \pi . 14^2$ cm^2

Volume of the frustum of cone

$$= \frac{height}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$$

$$= \frac{12}{3}\left[\pi . 22^2 + \pi . 14^2 + \pi . 22 \times 14\right]$$

$$= 4\pi\left[22^2 + 14^2 + 22 \times 14\right]$$

$$= 4\pi \times 988$$

$$= 12420.57 \text{ cm}^3$$

Amount of water $= \frac{12420.57}{1000}$ L $= 12.42057$ L

(5) What is the length of the greatest rod that can be placed in a room whose length is 30 ft, breadth is 24 ft and height 18 ft?

Soln.

① A room is internally 240 cm long, 180 cm broad and 144 cm high. Find the number of bricks measuring 9 cm x 4.5 cm x 3 cm required for the construction of its walls 18 cm thick.

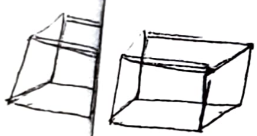
Surface area of the room = Area of the two ends + Area of side faces

$= 2 \times 240 \times 180 + 2(240+180)144$

$= 480 \times 180 + 288 \times 420$

$= 86400 + 120960$

$= 207360 \text{ cm}^2$



Surface area of the bricks $= 2 \times 9 \times 4.5 + 2(9+4.5)3$

$= 81 + 6 \times 13.5$

$= 81 + 81$

$= 162 \text{ cm}^2$

In 18 cm thick wall there will be two piece of brick

(15) (a) A cylindrical vessel has a base of diameter 10cm. It is completely filled by emptying into it the contents of a hemispherical bowl also of diameter 10cm. Find the height of the cylinder. (i)

Here

Volume of hemisphere = Volume of cylinder

$\Rightarrow \frac{1}{2}\left(\frac{4}{3}\pi R^3\right) = \pi R^2 \times h$

$\Rightarrow \frac{2}{3} \times \cancel{\frac{22}{7}} \times \left(\frac{10}{2}\right)^3 = \cancel{\pi} \times \left(\frac{10}{2}\right)^2 \times h$

$\Rightarrow \frac{2}{3} \times \frac{1000}{8} \times \frac{4}{100} = h$?

$\Rightarrow h = \frac{10}{3}$ cm

(b) An irregular plot has the following offsets measured from one end at end equal distance apart (ii)

x :	0	12	24	36	48	60	72	84	96	108	120
d :	53	52	47	49	53	63	58	61	52	49	48

Common distance = 12

Area $= \frac{12}{3}\left[(53+48)+2(47+53+58+52)+4(52+49+63+61+49)\right]$

$= 4\left[101+2\times 210+4\times 274\right]$

$= 4\left[101+420+1096\right]$

$= 4\times 1617 = 6468$ square unit