7. Properties of Matter

7.1 Interatomic Forces

The forces between the atoms due to electrostatic interaction between the charges of the atoms are called interatomic forces.

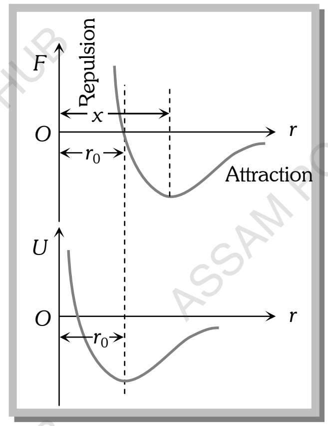
(1) When two atoms are brought close to each other to a distance of the order of 10^{-10} m, attractive interatomic force is produced between two atoms.

(2) This attractive force increases continuously with decrease in r and becomes maximum for one value of r called critical distance, represented by x (as shown in the figure).

(3) When the distance between the two atoms becomes r_0, the interatomic force will be zero. This distance r_0 is called normal or equilibrium distance.

(4) When the distance between the two atoms further decreased, the interatomic force becomes repulsive in nature and increases very rapidly.

(5) The potential energy U is related with the interatomic force F by the following relation.



$$F = \frac{-dU}{dr}$$

When the distance between the two atoms becomes r_0, the potential energy of the system of two atoms becomes minimum (*i.e.* attains maximum negative value hence the two atoms at separation r_0 will be in a state of equilibrium.

7.2 Intermolecular Forces

The forces between the molecules due to electrostatic interaction between the charges of the molecules are called intermolecular forces. These forces are also called Vander Waal forces and are quite weak as compared to inter-atomic forces.

7.3 Comparison between Interatomic and Intermolecular Forces

(i) Both the forces are electrical in origin.

(ii) Both the forces are active over short distances.

(iii) General shape of force-distance graph is similar for both the forces.

(iv) Both the forces are attractive up to certain distance between atoms/molecules and become repulsive when the distance between them become less than that value.

7.4 Solids

A solid is that state of matter in which its constituent atoms or molecules are held strongly at the position of minimum potential energy and it has a definite shape and volume.

7.5 Elastic Property of Matter

(1) ***Elasticity:*** The property of matter by virtue of which a body tends to regain its original shape and size after the removal of deforming force is called elasticity.

(2) ***Plasticity:*** The property of matter by virtue of which it does not regain its original shape and size after the removal of deforming force is called plasticity.

(3) ***Perfectly elastic body:*** If on the removal of deforming forces the body regain its original configuration completely it is said to be perfectly elastic.
A quartz fibre and phosphor is the nearest approach to the perfectly elastic body.

(4) ***Perfectly plastic body:*** If the body does not have any tendency to recover its original configuration, on the removal of deforming force, it is said to be perfectly plastic.
Paraffin wax, wet clay are the nearest approach to the perfectly plastic body.
Practically there is no material which is either perfectly elastic or perfectly plastic.

(5) ***Reason of elasticity:*** On applying the deforming forces, restoring forces are developed. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium position ($r = r_0$) and hence the body regains its original form.

(6) ***Elastic limit:*** The maximum deforming force upto which a body retains its property of elasticity is called elastic limit of the material of body.
Elastic limit is the property of a body whereas elasticity is the property of material of the body.

(7) ***Elastic fatigue:*** The temporary loss of elastic properties because of the action of repeated alternating deforming force is called elastic fatigue.
It is due to this reason:
(i) Bridges are declared unsafe after a long time of their use.
(ii) Spring balances show wrong readings after they have been used for a long time.
(iii) We are able to break the wire by repeated bending.

(8) ***Elastic after effect:*** The time delay in which the substance regains its original condition after the removal of deforming force is called elastic after effect. it is negligible for perfectly elastic substance, like quartz, phosphor bronze and large for glass fibre.

7.6 Stress

The internal restoring force acting per unit area of cross section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

Unit: N/m^2 (S.I.), $dyne/cm^2$ (C.G.S.)
Stress developed in a body depends upon how the external forces are applied over it.
On this basis there are two types of stresses: Normal and Shear or tangential stress

(1) ***Normal stress:*** Here the force is applied normal to the surface.
It is again of two types: Longitudinal and Bulk or volume stress
(i) *Longitudinal stress*

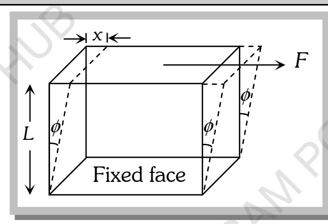
(a) Deforming force is applied parallel to the length and causes increase in length.
(b) Area taken for calculation of stress is area of cross section.
(c) Longitudinal stress produced due to increase in length of a body under a deforming force is called tensile stress.
(d) Longitudinal stress produced due to decrease in length of a body under a deforming force is called compressional stress.

(ii) *Bulk or Volume stress*
(a) It occurs in solids, liquids or gases.
(b) Deforming force is applied normal to surface at all points.
(c) It is equal to change in pressure because change in pressure is responsible for change in volume.

(2) ***Shear or tangential stress:*** It comes in picture when successive layers of solid move on each other *i.e.* when there is a relative displacement between various layers of solid.
(i) Here deforming force is applied tangential to one of the faces.
(ii) Area for calculation is the area of the face on which force is applied.
(iii) It produces change in shape, volume remaining the same.

7.7 Strain

The ratio of change in configuration to the original configuration is called strain. It has no dimensions and units. Strain are of three types:



(1) ***Linear strain:*** Linear strain = $\frac{\text{Change in length}(\Delta l)}{\text{Original length}(l)}$

Linear strain in the direction of deforming force is called longitudinal strain and in a direction perpendicular to force is called lateral strain.

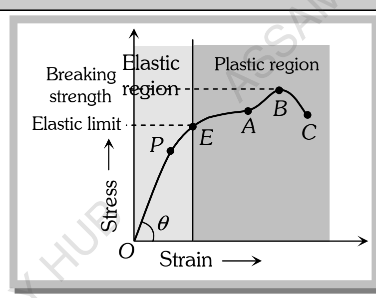
(2) ***Volumetric strain:*** Volumetric strain = $\frac{\text{Change in volume}(\Delta V)}{\text{Original volume}(V)}$

(3) ***Shearing strain:*** It is defined as angle in radians through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of tangential force.

$$\phi = \frac{x}{L}$$

❑ When a beam is bent both compression strain as well as an extension strain is produced.

7.8 Stress-strain Curve



(1) When the strain is small (region OP) stress is proportional to strain. This is the region where the so called Hooke's law is obeyed. The point P is called limit of proportionality and slope of line OP gives the

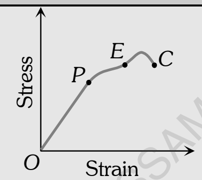
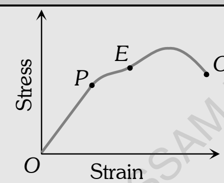
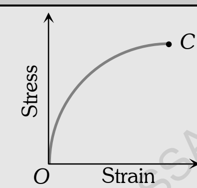
Young's modulus Y of the material of the wire. $Y = \tan\theta$.

(2) Point E known as elastic limit or yield-point.

(3) Between EA, the strain increases much more.

(4) The region $EABC$ represents the plastic behaviour of the material of wire.

(5) Stress-strain curve for different materials.

Brittle material	Ductile material	Elastomers
		
The plastic region between E and C is small for brittle material and it will break soon after the elastic limit is crossed.	The material of the wire have a good plastic range and such materials can be easily changed into different shapes and can be drawn into thin wires.	for elastomers and strain produced is much larger than the stress applied. Such materials have no plastic range and the breaking point lies very close to elastic limit. Example rubber.

7.9 Hooke's law and Modulus of Elasticity

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain or $\frac{\text{stress}}{\text{strain}} = \text{constant} = E$

The constant E is called modulus of elasticity.

(1) It's value depends upon the nature of material of the body and the manner in which the body is deformed.

(2) It's value depends upon the temperature of the body.

(3) It's value is independent of the dimensions of the body.

There are three modulii of elasticity namely Young's modulus (Y), Bulk modulus (K) and modulus of rigidity (η) corresponding to three types of the strain.

7.10 Young's Modulus (Y)

It is defined as the ratio of normal stress to longitudinal strain within limit of proportionality.

$$Y = \frac{\text{Normal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L} = \frac{FL}{Al}$$

Thermal stress: If a rod is fixed between two rigid supports, due to change in temperature its length will change and so it will exert a normal. This stress is called thermal stress. Thermal stress = $Y\alpha\Delta\theta$ force produced in the body = $YA\alpha\,\Delta\theta$

7.11 Work Done in Stretching a Wire

In stretching a wire work is done against internal restoring forces. This work is stored in the wire as elastic potential energy or strain energy.

$\therefore$ Energy stored in wire $U = \frac{1}{2}\frac{YAl^2}{L} = \frac{1}{2}Fl$

Energy stored in per unit volume of wire.

$$= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2} \times Y \times (\text{strain})^2 = \frac{1}{2Y}(\text{stress})^2$$

7.12 Breaking of Wire

When the wire is loaded beyond the elastic limit, then strain increases much more rapidly. The maximum stress corresponding to B (see stress-strain curve) after which the wire begin to flow and breaks, is called breaking stress or tensile strength and the force by application of which the wire breaks is called the breaking force.

(i) Breaking force depends upon the area of cross-section of the wire

(ii) Breaking stress is a constant for a given

(iii) Breaking force is independent of the length of wire.

(iv) Breaking force $\propto \pi r^2$.

(v) Length of wire if it breaks by its own weight. $L = \frac{P}{dg}$

7.13 Bulk Modulus

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus.

This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}; \qquad K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The reciprocal of bulk modulus is called compressibility.

C = compressibility = $\frac{1}{K} = \frac{\Delta V}{pV}$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}\,cm^2$.

Gases have two bulk moduli, namely isothermal elasticity E_θ and adiabatic elasticity E_ϕ.

7.14 Density of Compressed Liquid

If a liquid of density ρ, volume V and bulk modulus K is compressed, then its density increases.

$$\rho' = \rho\left[1 + \frac{\Delta P}{K}\right] = \rho[1 + C\Delta P] \qquad \left[\text{As } \frac{1}{K} = C\right]$$

7.15 Modulus of Rigidity

Within limits of proportionality, the ratio of tangential stress to the shearing strain is called modulus of rigidity of the material of the body and is denoted by η, *i.e.*

$$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\phi} = \frac{F}{A\phi}$$

Only solids can exhibit a shearing as these have definite shape.

7.16 Poisson's Ratio

Lateral strain: The ratio of change in radius to the original radius is called lateral strain.

Longitudinal strain: The ratio of change in length to the original length is called longitudinal strain.

The ratio of lateral strain to longitudinal strain is called Poisson's ratio (σ).

i.e. $$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

7.17 Relation Between Volumetric Strain, Lateral Strain and Poisson's Ratio

$$\sigma = \frac{1}{2}\left[1 - \frac{dV}{AdL}\right]$$ [where A = cross-section of bar]

(i) If a material having $\sigma = -0.5$ then
Volume = constant *i.e.,* the material is incompressible.

(ii) Theoretical value of Poisson's ratio $-1 < \sigma < 0.5$.

(iii) Practical value of Poisson's ratio $0 < \sigma < 0.5$.

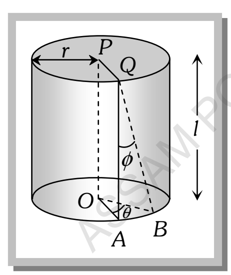
7.18 Relation between *Y*, *k*, η and σ

$Y = 3K(1 - 2\sigma)$ and $Y = 2\eta(1 + \sigma)$; $Y = \dfrac{9K\eta}{3K + \eta}$ and $\sigma = \dfrac{3K - 2\eta}{6K + 2\eta}$

7.19 Torsion of Cylinder

If the upper end of a cylinder is clamped and a torque is applied at the lower end the cylinder gets twisted by angle θ. Simultaneously shearing strain ϕ is produced in the cylinder.

(i) The angle of twist θ is directly proportional to the distance from the fixed end of the cylinder.



(ii) The value of angle of shear ϕ is directly proportional to the radius of the cylindrical shell.

(iii) Relation between angle of twist (θ) and angle of shear (ϕ)

$$AB = r\theta = \phi l \qquad \therefore\ \phi = \frac{r\theta}{l}$$

(iv) Twisting couple per unit twist or torsional rigidity or torque required to produce unit twist.

$$C = \frac{\pi\eta r^4}{2l} \qquad \therefore\ C \propto r^4 \propto A^2$$

(v) Work done in twisting the cylinder through an angle θ is $W = \frac{1}{2}C\theta^2 = \frac{\pi\eta r^4\theta^2}{4l}$

7.20 Elastic Hysteresis

Hysteresis loop: The area of the stress-strain curve is called the hysteresis loop and it is numerically equal to the work done in loading the material and then unloading it.

7.21 Factors Affecting Elasticity

(1) ***Hammering and rolling:*** This result in increase in the elasticity of material.
(2) ***Annealing:*** Annealing results in decrease in the elasticity of material.
(3) ***Temperature:*** Elasticity decreases with rise in temperature but the elasticity of invar steel (alloy) does not change with change of temperature.
(4) ***Impurities:*** The type of effect depends upon the nature of impurities present in the material.

7.22 Important Facts about Elasticity

(1) The body which requires greater deforming force to produce a certain change in dimension is more elastic.
(2) When equal deforming force is applied on different bodies then the body which shows less deformation is more elastic.
  (i) Water is more elastic than air as volume change in water is less for same applied pressure.
  (ii) Four identical balls of different materials are dropped from the same height then after collision balls rises upto different heights.
  $h_{\text{ivory}} > h_{\text{steel}} > h_{\text{rubber}} > h_{\text{clay}}$ because $Y_{\text{ivory}} > Y_{\text{steel}} > Y_{\text{rubber}} > Y_{\text{clay}}$.
(3) For a given material there can be different moduli of elasticity depending on the type of stress and strain.
(4) $K_{\text{solid}} > K_{\text{liquid}} > K_{\text{gas}}$
(5) Elasticity of a rigid body is infinite.

7.23 Practical Applications of Elasticity

(i) The thickness of the metallic rope used in the crane is decided from the knowledge of elasticity.
(ii) Maximum height of a mountain on earth can be estimated.
(iii) A hollow shaft is stronger than a solid shaft made of same mass, length and material.

7.24 Intermolecular Force

The force of attraction or repulsion acting between the molecules are known as intermolecular force. The nature of intermolecular force is electromagnetic.

The intermolecular forces of attraction may be classified into two types.

Cohesive force	Adhesive force
The force of attraction between molecules of same substance is called the force of cohesion. This force is lesser in liquids and least in gases.	The force of attraction between the molecules of the different substances is called the force of adhesion.

7.25 Surface Tension

The property of a liquid due to which its free surface tries to have minimum surface area is called surface tension. A small liquid drop has spherical shape due to surface tension.Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid. then $T = (F/L)$.

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

(3) Dimension: $[MT^{-2}]$. (Similar to force constant)

(4) Units: N/m (S.I.) and $Dyne/cm$ [C.G.S.]

7.26 Factors Affecting Surface Tension

(1) ***Temperature:*** The surface tension of liquid decreases with rise of temperature

$$T_t = T_0(1 - \alpha t)$$

where T_t, T_0 are the surface tensions at $t°C$ and 0°C respectively and α is the temperature coefficient of surface tension.

(2) ***Impurities:*** A highly soluble substance like sodium chloride when dissolved in water, increases the surface tension of water. But the sparingly soluble substances like phenol when dissolved in water, decreases the surface tension of water.

7.27 Surface Energy

The potential energy of surface molecules per unit area of the surface is called surface energy.

Unit: $Joule/m^2$ (S.I.) erg/cm^2 (C.G.S.)

Dimension: $[MT^{-2}]$

$\therefore\ W = T \times \Delta A$ [ΔA = Total increase in area of the film from both the sides]

i.e. surface tension may be defined as the amount of work done in increasing the area of the liquid surface by unity against the force of surface tension at constant temperature.

7.28 Work Done in Blowing a Liquid Drop or Soap Bubble

(1) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then

$W = T \times$ Increment in surface area

$W = T \times 4\pi[r_2^2 - r_1^2]$ [drop has only one free surface]

(2) In case of soap bubble

$W = T \times 8\pi[r_2^2 - r_1^2]$ [Bubble has two free surfaces]

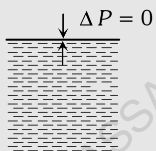
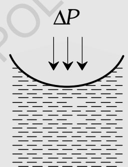
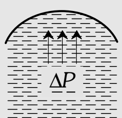
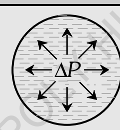
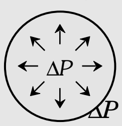
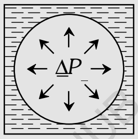
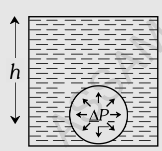
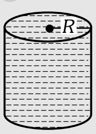
7.29 Splitting of Bigger Drop

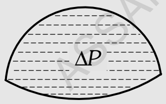
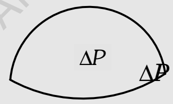
When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. $R^3 = nr^3$

Work done = $T \times \Delta A = T$ [Total final surface area of n drops – surface area of big drop] = $T[n^4\pi r^2 - 4\pi R^2]$.

7.30 Excess Pressure

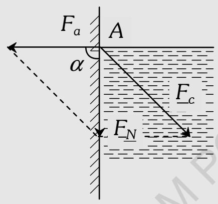
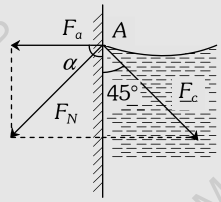
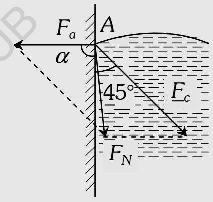
Excess pressure in different cases is given in the following table:

Plane surface	Concave surface
 $\Delta P = 0$	 $\Delta P = \frac{2T}{R}$
Convex surface	**Drop**
 $\Delta P = \frac{2T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble in air	**Bubble in liquid**
 $\Delta P = \frac{4T}{R}$	 $\Delta P = \frac{2T}{R}$
Bubble at depth *h* below the free surface of liquid of density *d*	**Cylindrical liquid surface**
 $\Delta P = \frac{2T}{R} + hdg$	 $\Delta P = \frac{T}{R}$

Liquid surface of unequal radii	Liquid film of unequal radii
 $\Delta P = T\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$	 $\Delta P = 2T\left[\frac{1}{R_1}+\frac{1}{R_2}\right]$

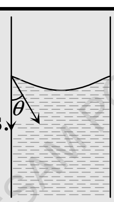
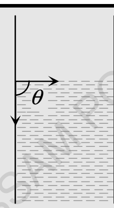
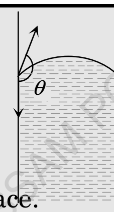
7.31 Shape of Liquid Meniscus

The curved surface of the liquid is called meniscus of the liquid.

If $F_c = \sqrt{2}Fa$ $\tan\alpha = \infty \therefore \alpha = 90^o$ *i.e.* the resultant force acts vertically downwards. Hence the liquid meniscus must be horizontal.	$F_c < \sqrt{2}Fa$ tan α = positive $\therefore$ α is acute angle *i.e.* the resultant force directed outside the liquid. Hence the liquid meniscus must be concave upward.	$F_c > \sqrt{2}Fa$ tan α = negative $\therefore$ α is obtuse angle *i.e.* the resultant force directed inside the liquid. Hence the liquid meniscus must be convex upward.
		
Example: Pure water in silver coated capillary tube.	*Example:* Water in glass capillary tube.	*Example:* Mercury in glass capillary tube.

7.32 Angle of Contact

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

$\theta < 90^o$; $F_a > \frac{F_c}{\sqrt{2}}$ concave meniscus. Liquid wets the solid surface. 	$\theta = 90^o$; $F_a = \frac{F_c}{\sqrt{2}}$ plane meniscus. Liquid does not wet the solid surface. 	$\theta > 90^o$; $F_a < \frac{F_c}{\sqrt{2}}$ convex meniscus. Liquid does not wet the solid surface. 

(i) Its value lies between 0^o and 180^o
$\theta = 0°$ for pure water and glass, $\theta = 90°$ for water and silver
(ii) On increasing the temperature, angle of contact decreases.

(iii) Soluble impurities increases the angle of contact.
(iv) Partially soluble impurities decreases the angle of contact.

7.33 Capillarity

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity.

The cause of capillarity is the difference in pressures on two sides curved surface of liquid.

7.34 Ascent Formula

When one end of capillary tube of radius r is immersed into a liquid of density d which wets the sides of the capillary R = radius of curvature of liquid meniscus.

T = surface tension of liquid

P = atmospheric pressure

$$\therefore \quad h = \frac{2T\cos\theta}{rdg}$$

IMPORTANT POINTS

(i) The capillary rise depends on the nature of liquid and solid both *i.e.* on T, d, θ and R.

(ii) Capillary action for various liquid-solid pair.

Meniscus	Angle of contact	Level
Concave	$\theta < 90^o$	Rises
Plane	$\theta = 90^o$	No rise no fall
Convex	$\theta > 90^o$	Fall

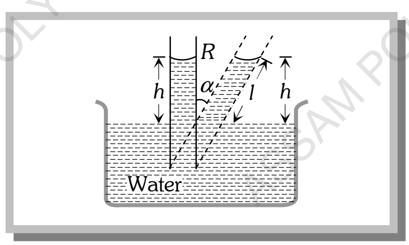
(iii) Lesser the radius of capillary greater will be the rise and vice-versa. This is called Jurin's law.

(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by $h = \frac{2T\cos\theta}{rdg} - \frac{r}{3}$

(v) In case of capillary of insufficient length, *i.e.*, $L < h$, the liquid will neither overflow from the upper end. The liquid after reaching the upper end will increase the radius of its meniscus without changing nature such that:

$$hr = Lr' \because L < h \qquad \therefore r' > r$$

(vi) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.



$$h = l\cos\alpha \text{ or } l = \frac{h}{\cos\alpha}$$

7.35 Pressure

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is $P = F/A$ $P = F/A$

(1) ***Units:*** N/m^2 or Pascal (S.I.) and Dyne/cm^2 (C.G.S.)

(2) ***Dimension:*** $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

(3) ***Pressure is a tensor quantity.***

(4) ***Atmospheric pressure:*** 1 atm = $1/01 \times 10^5$ *Pa* = 1.01 bar = torr

(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P = P_0 + h\rho g$.

(6) ***Gauge pressure:*** The pressure difference between hydrostatic pressure P and atmospheric pressure P_0 is called gauge pressure. $P - P_0 + h\rho g$

7.36 Density

In a fluid, at a point, density ρ is defined as: $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

(1) It has dimensions $[ML^{-3}]$ and S.I. unit kg/m^3 while C.G.S. unit *g/cc* with 1 g/cc = $10^3 kg/m^3$.

(2) Relative density or specific gravity which is defined as: $RD = \frac{\text{Density of body}}{\text{Density of water}}$

(3) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\sum m_i}{\sum (m_i / p_i)}$$

(4) With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, $\rho \simeq \rho_0 (1 - \gamma \Delta\theta)$

(5) With increase in pressure due to decrease in volume, density will increase.

$\rho \simeq \rho_0 \left(1 + \frac{\Delta p}{B}\right)$ where B is blk modulus.

7.37 Pascal's Law

The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Example: Hydraulic lift, hydraulic press and hydraulic brakes.

7.38 Archimedes Principle

When a body is immersed partly or wholly in a fluid, in rest it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called Archimedes principle.

Apparent weight of the body of density (ρ) when immersed in a liquid of density (σ).

Apparent weight = Actual weight – Upthrust = $W - F_{up} = V\rho g - V\sigma g = V(\rho - \sigma)g = V\rho g\left(1 - \frac{\sigma}{\rho}\right)$

$$\therefore \qquad W_{APP} = W\left(1 - \frac{\sigma}{\rho}\right)$$

(1) Relative density of a body (R.D.) = $\frac{\text{Weight of body in air}}{\text{Weight in air–weight in water}} = \frac{W_1}{W_1 - W_2}$

(2) If the loss of weight of a body in water is 'a' while in liquid is 'b'

$$\therefore \qquad \frac{\sigma_L}{\sigma_W} = \frac{\text{Upthrust on body in liquid}}{\text{Upthrust on body in water}} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{a}{b} = \frac{W_{\text{air}} - W_{\text{liquid}}}{W_{\text{air}} - W_{\text{water}}}$$

7.39 Floatation

(1) ***Translatory equilibrium:*** When a body of density ρ and volume V is immersed in a liquid of density ρ, the forces acting on the body are Weight of body $W = mg = V\rho g$, acting vertically downwards through centre of gravity of the body.
Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.
  - (i) If density of body is greater than that of liquid $\rho > \sigma$
    Weight will be more than upthrust so the body will sink
  - (ii) If density of body is equal to that of liquid $\rho = \sigma$
    Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium anywhere in the liquid.
  - (iii) If density of body is lesser than that of liquid $\rho < \sigma$
    Weight will be less than upthrust so the body will move upwards and in equilibrium will float partially immersed in the liquid $V\rho g = V_{in}\sigma g$ where V_{in} is the volume of body in the liquid.

(2) ***Rotatory Equilibrium:*** When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta-centre. If the meta-centre M is above the centre of gravity then object remains in stable equilibrium.However, if meta-centre goes below centre of gravity then object remains in stable equilibrium.

7.40 Streamline, Laminar and Turbulent Flow

(1) ***Stream line flow:*** Stream line flow of a liquid is that flow in which each element of the liquid passing through a point travels along the same path and with the same velocity as the preceeding element passes through that point.
The two streamlines cannot cross each other and the greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place.

(2) ***Laminar flow:*** If a liquid is flowing over a horizontal surface with a steady flow and moves in the form of layers of different velocities which do not mix with each other, then the flow of liquid is called laminar flow.
In this flow the velocity of liquid flow is always less than the critical velocity of the liquid.

(3) ***Turbulent flow:*** When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disordered or irregular. Such a flow is called a turbulent flow.

7.41 Critical Velocity and Reynold's Number

The critical velocity is that velocity of liquid flow upto which its flow is streamlined and above which its flow becomes turbulent. Reynold's number is a pure number which determines the nature of flow of liquid through a pipe.

It is defined as the ratio of the inertial force per unit area to the viscous force per unit area for a flowing fluid.

So by the definition of Reynolds number $N_R = \frac{\text{Inertial force per unit area}}{\text{Viscous force per unit area}} = \frac{v^2\rho}{\eta v / r} = \frac{v\rho r}{\eta}$

If the value of Reynold's number:

(i) Lies between 0 to 2000, the flow of liquid is streamline or laminar.

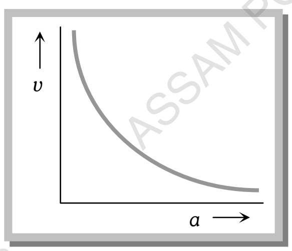
(ii) Lies between 2000 to 3000, the flow of liquid is unstable changing from streamline to turbulent flow.

(iii) Above 3000, the flow of liquid is definitely turbulent.

7.42 Equation of Continuity

The equation of continuity is derived from the principle of conservation of mass.

For an incompressible, streamlined and non-viscous liquid product of area of cross section of tube and velocity of liquid remains constant.



i.e. $a_1v_1 = a_2v_2$

or av = constant; or $a \propto \frac{1}{v}$

When water falls from a tap, the velocity of falling water under the action of gravity will increase with distance from the tap (*i.e,* $v_2 > v_1$). So in accordance with continuity equation the cross section of the water stream will decrease (*i.e.,* $A_2 < A_1$), *i.e.,* the falling stream of water becomes narrower.

7.43 Energy of a Flowing Fluid

Pressure Energy	Potential energy	Kinetic energy
It is the energy possessed by a liquid by virtue of its pressure. It is the measure of work done in pushing the liquid against pressure	It is the energy possessed by liquid by virtue of its height or position above the surface of earth or any	It is the energy possessed by a liquid by virtue of its motion or velocity.

without imparting any velocity to it.	reference level taken as zero level.	
Pressure energy of the liquid PV	Potential energy of the liquid mgh	Kinetic energy of the liquid $\frac{1}{2}mv^2$
Pressure energy per unit mass of the liquid $\frac{P}{\rho}$	Potential energy per unit mass of the liquid gh	Kinetic energy per unit mass of the liquid $\frac{1}{2}v^2$
Pressure energy per unit volume of the liquid P	Potential energy per unit volume of the liquid ρgh	Kinetic energy per unit volume of the liquid $\frac{1}{2}\rho v^2$

7.44 Bernoulli's Theorem

According to this theorem the total energy (pressure energy, potential energy and kinetic energy) per unit volume or mass of an incompressible and non-viscous fluid in steady flow through a pipe remains constant throughout the flow.

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

(i) Bernoulli's theorem for unit mass of: $\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{constant}$

(ii) Dividing above equation by g we get $\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$

Here $\frac{P}{\rho g}$ is called pressure head, h is called gravitational head and $\frac{v^2}{2g}$ is called velocity head.

7.45 Applications of Bernoulli's Theorem

(i) ***Attraction between two closely parallel moving boats***

(ii) ***Working of an aeroplane:*** 'dynamic lift' (= pressure difference × area of wing)

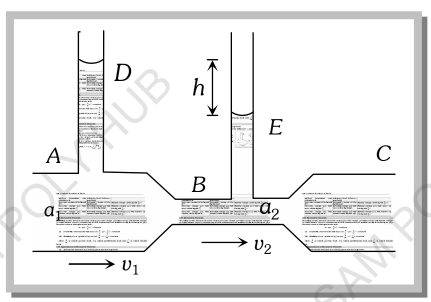
(iii) ***Action of atomiser:***

(iv) ***Blowing off roofs by wind storms***

(v) ***Magnus effect:*** When a spinning ball is thrown, it deviates from its usual path in flight. This effect is called Magnus effect.

(vi) ***Venturimeter:*** It is a device used for measuring the rate of flow of liquid through pipes.

Rate of flow of liquid $V = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}}$



7.46 Velocity of Efflux

Velocity of efflux from a hole made at a depth h below the free surface of the liquid(of depth H) is given by $v = \sqrt{2gh}$.

Which is same as the final speed of a free falling object from rest through a distance h. This result is known as Torricelli's theorem.

(i) Time taken by the liquid to reach the base-level $t = \sqrt{\dfrac{2(H-h)}{g}}$

(ii) Horizontal range (x): $x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$

For maximum range $\dfrac{dx}{dh} = 0 \;\; \therefore \;\; h = \dfrac{H}{2}$

$\therefore$ Maximum range $x_{max} = 2\sqrt{\dfrac{H}{2}\left[H - \dfrac{H}{2}\right]} = H$

7.47 Viscosity and Newton's law of Viscous Force

The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion is called viscous force.

Viscous force F is proportional to the area of the plane A and the velocity gradient $\dfrac{dv}{dx}$ in a direction normal to the layer,

i.e., $F = -\eta A \dfrac{dv}{dx}$

Where η is a constant called the coefficient of viscosity. Negative sign is employed because viscous force acts in a direction opposite to the flow of liquid.

(1) Units: dyne-s-cm^{-2} or Poise (C.G.S. system);
Newton-s-m^{-2} or Poiseuille or decapoise (S.I. system)
1 Poiseuille = 1 decapoise = 10 Poise

(2) Dimension: $[ML^{-1}T^{-1}]$

(3) With increase in pressure, the viscosity of liquids (except water) increases while that of gases is independent of pressure. The viscosity of water decreases with increase in pressure.

(4) Solid friction is independent of the area of surfaces in contact and the relative velocity between them.

(5) Viscosity represents transport of momentum, while diffusion and conduction represents transport of mass and energy respectively.

(6) The viscosity of gases increases with increase of temperature.

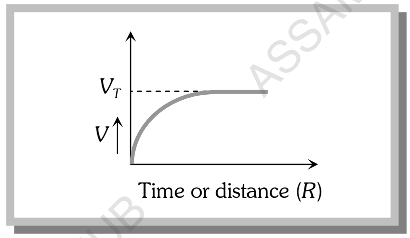
(7) The viscosity of liquid decreases with increase of temperature.

7.48 Stoke's Law and Terminal Velocity

Stokes established that if a sphere of radius r moves with velocity v through a fluid of viscosity η, the viscous force opposing the motion of the sphere is

$$F = 6\pi\eta r v \text{ (Stokes law)}$$

If a spherical body of radius r is dropped in a viscous fluid, it is first accelerated and then it's acceleration becomes zero and it attains a constant velocity called terminal velocity.



Terminal velocity $v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta}$

(i) If $\rho > \sigma$ then body will attain constant velocity in downward direction.
(ii) If $\rho < \sigma$ then body will attain constant velocity in upward direction.
Example: Air bubble in a liquid and clouds in sky.
(iii) Terminal velocity graph:

7.49 Poiseuille's Formula

Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l ' and radius r, then the volume of liquid coming out of the tube per second is

$$V = \frac{\pi P r^4}{8\eta l} \text{ (Poiseulle's equation)}$$

This equation also can be written as, $V = \frac{P}{R}$ where $R = \frac{8\eta l}{\pi r^4}$

R is called as liquid resistance.

(1) ***Series combination of tubes:***
  (i) The volume of liquid flowing through both the tubes *i.e.* rate of flow of liquid is same.
  (ii) Effective liquid resistance in series combination $R_{eff} = R_1 + R_2$

(2) ***Parallel combination of tubes:***
  (i) Pressure difference across both tubes remains same.
  (ii) Effective liquid resistance in parallel combination $\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$